

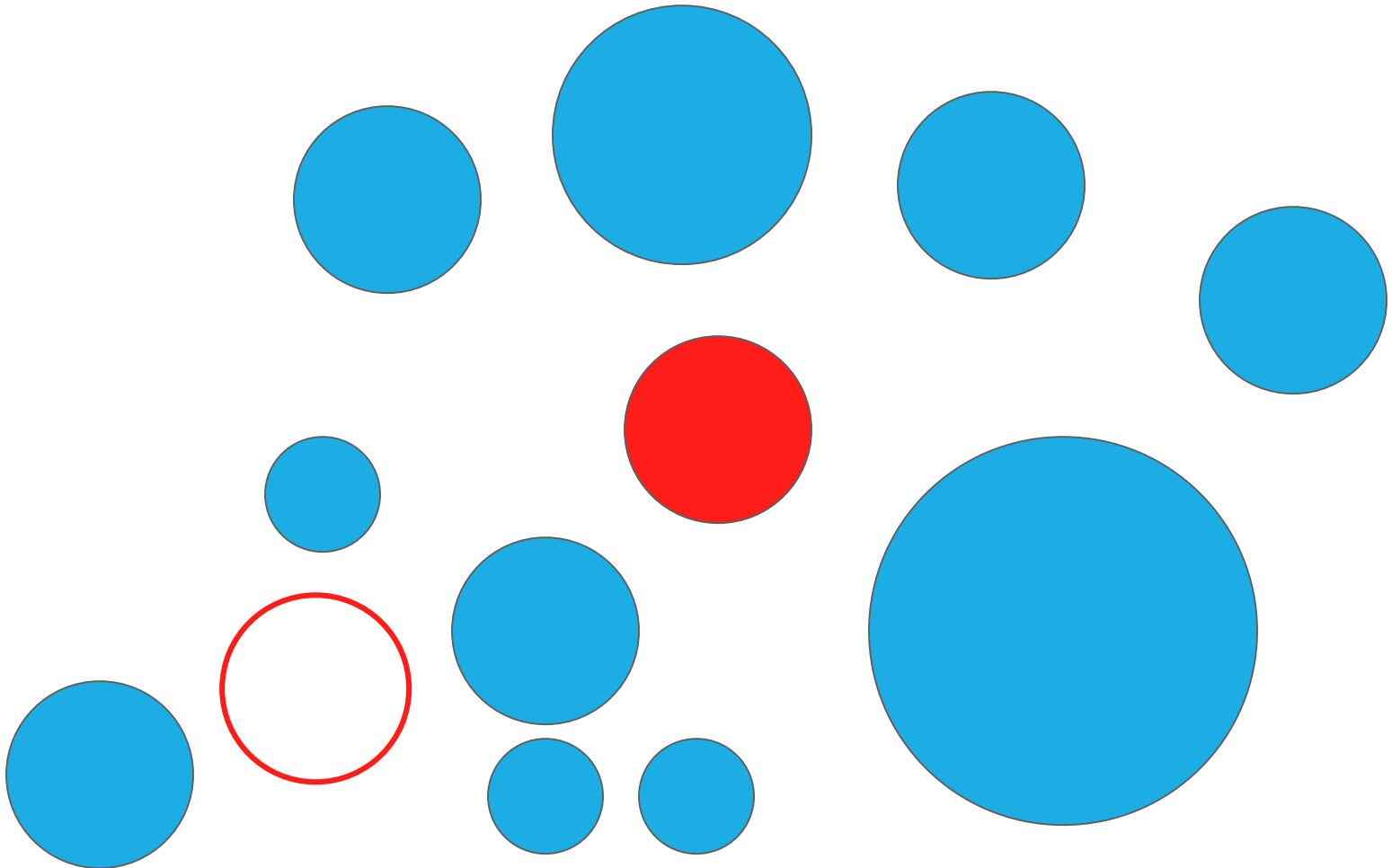
Algorithmic Robotics and Motion Planning

The Roomba in the café
Combinatorics and algorithms

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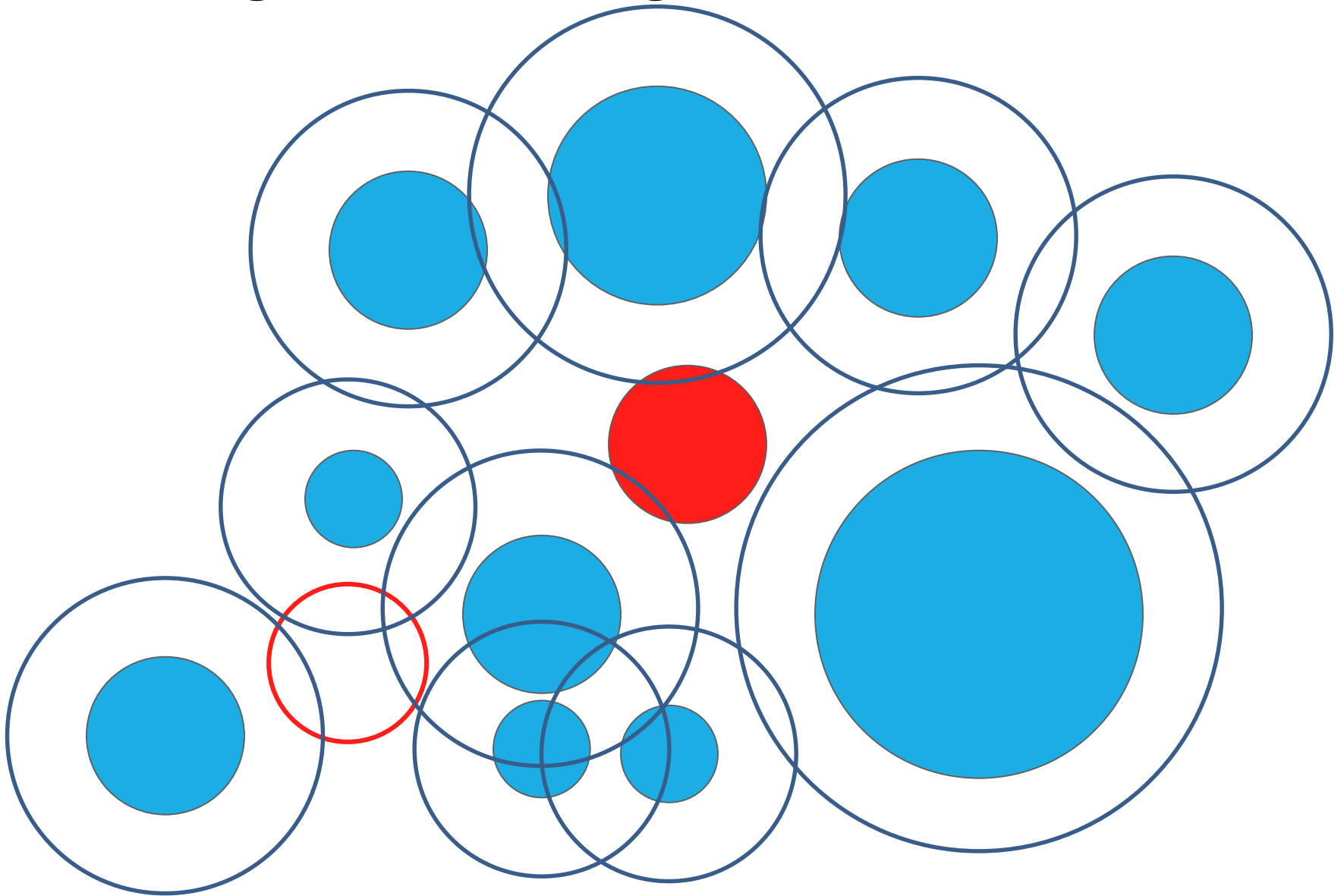
Moving a disc among discs



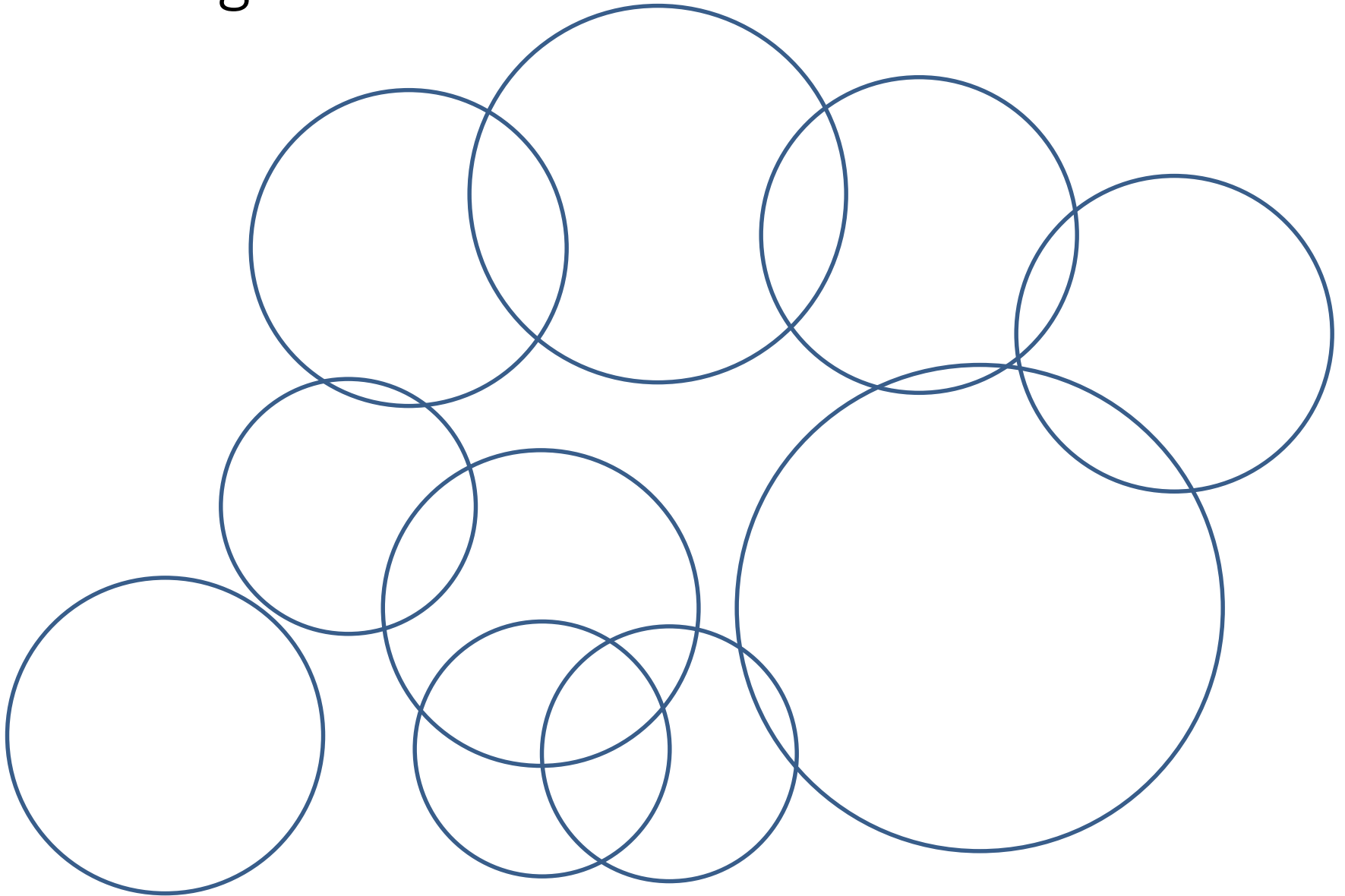
Outline

- the C-space
- combinatorial complexity
- representation
- algorithm
- algebra

Moving a disc among discs: C-obstacles



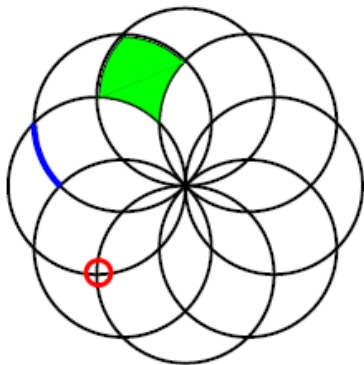
Arrangement of circles



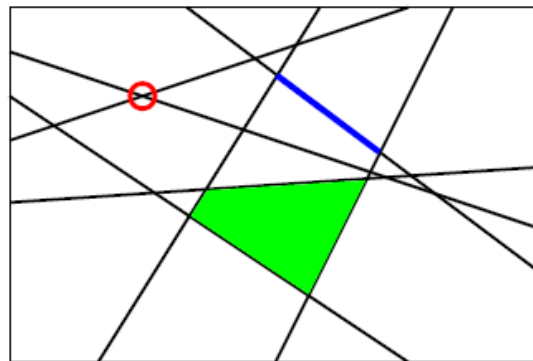
Arrangements (take I)

Definition (Arrangement)

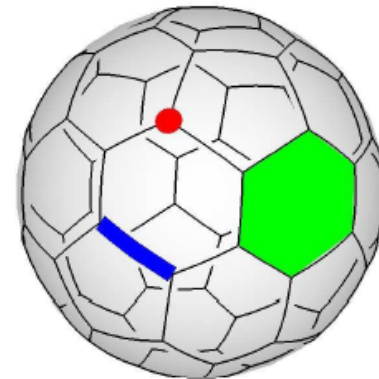
Given a collection \mathcal{C} of curves on a surface, the **arrangement** $\mathcal{A}(\mathcal{C})$ is the partition of the surface into **vertices**, **edges** and **faces** induced by the curves of \mathcal{C} .



An arrangement of circles in the plane.

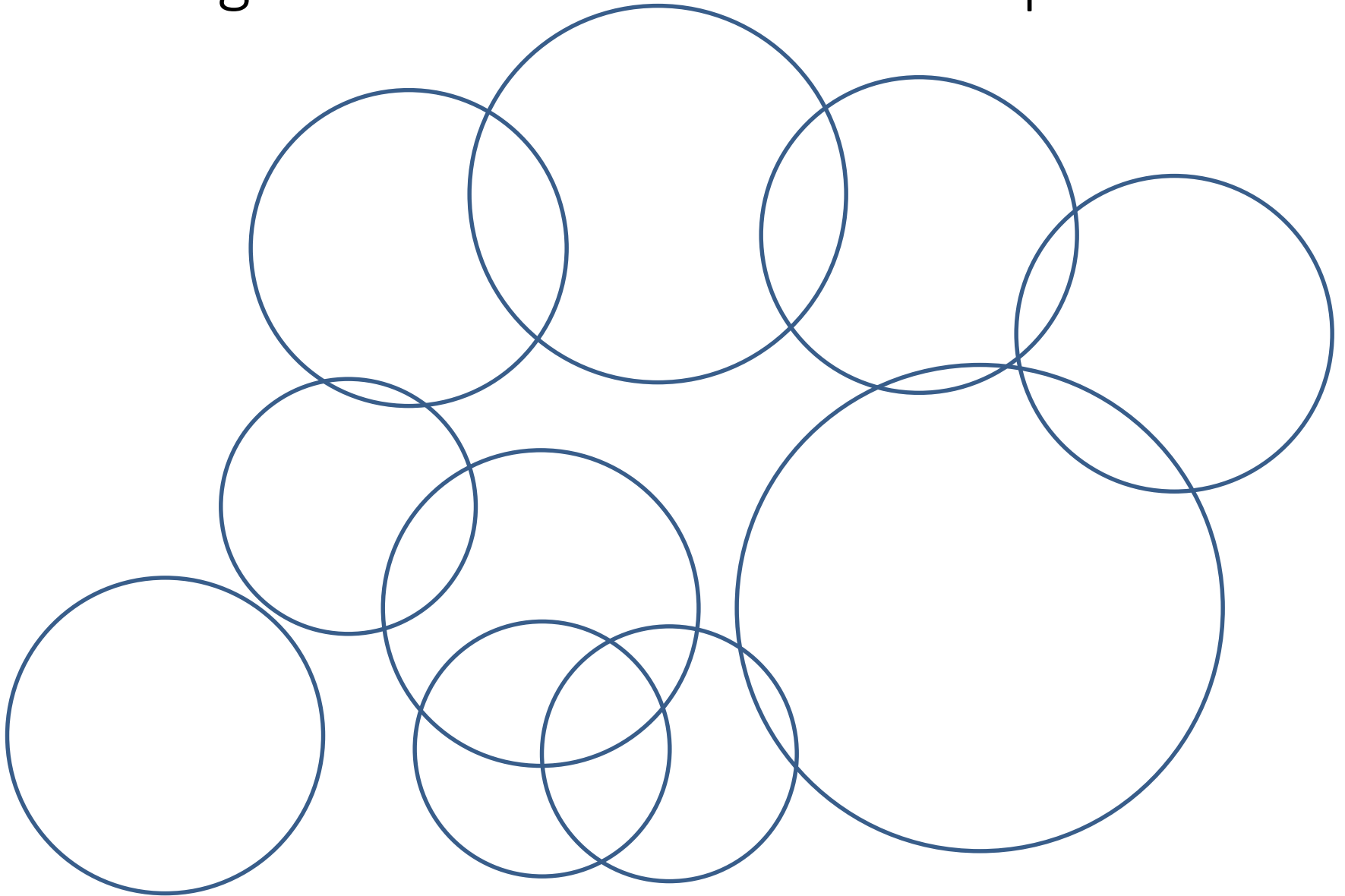


An arrangement of lines in the plane.

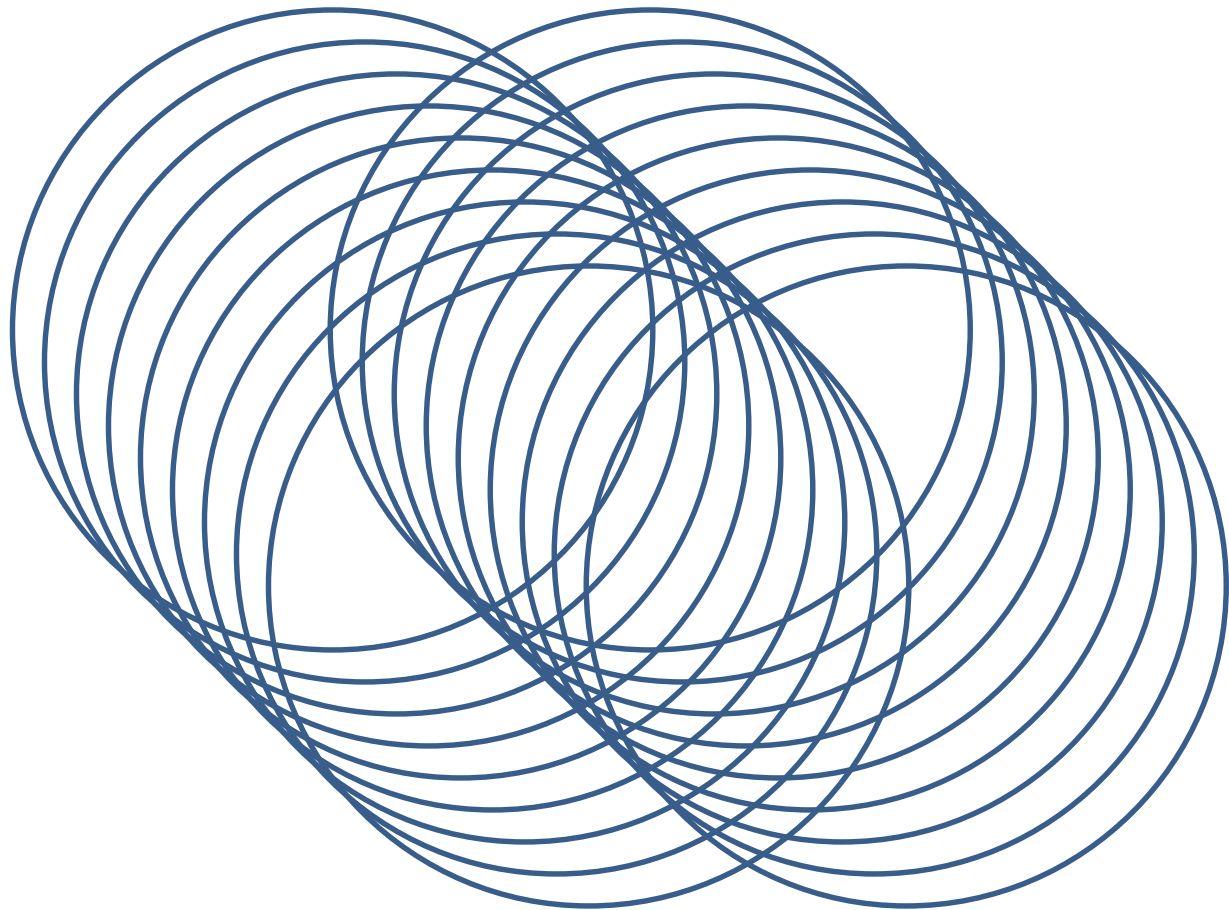


An arrangement of great-circle arcs on a sphere.

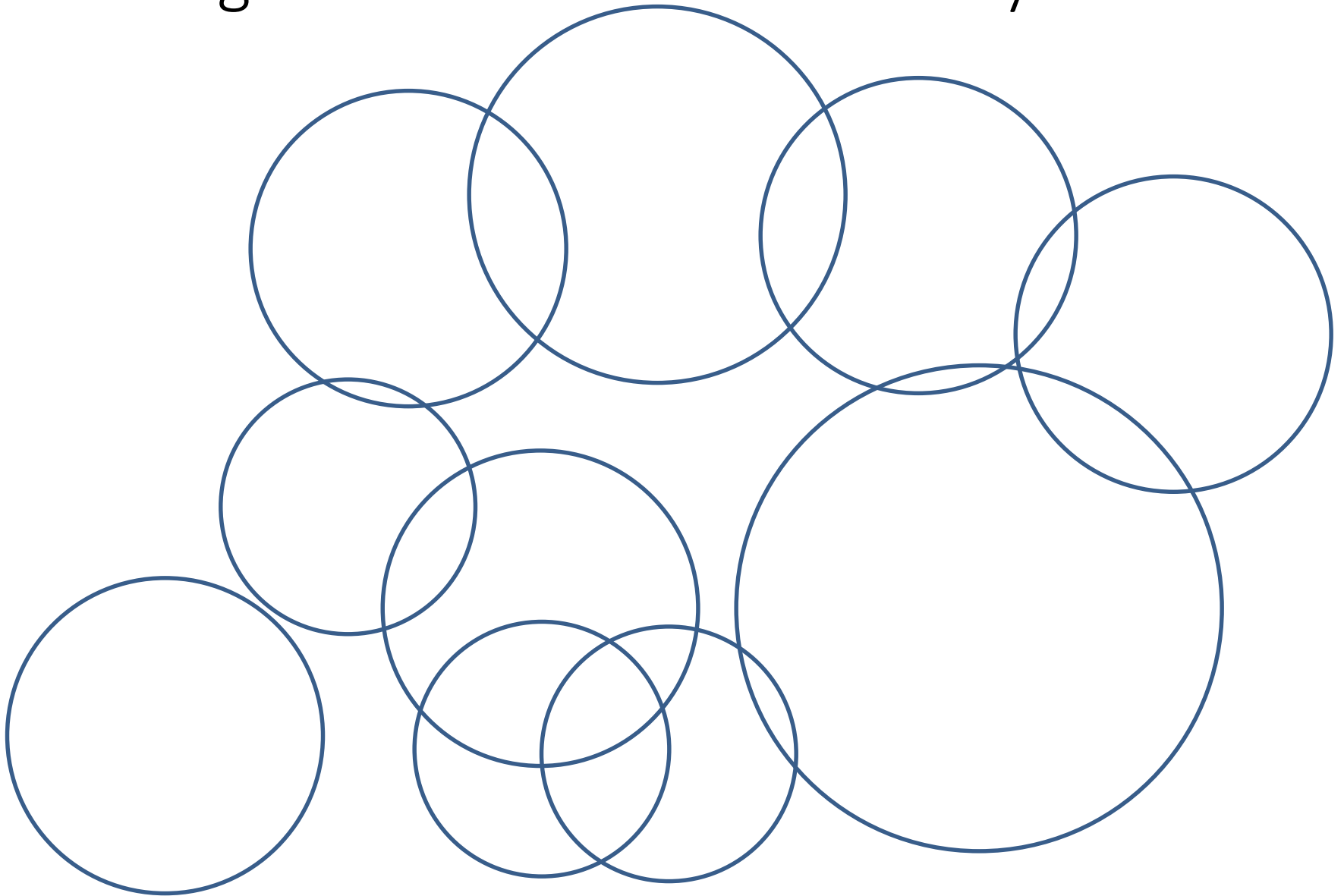
Arrangement of circles: how complex?



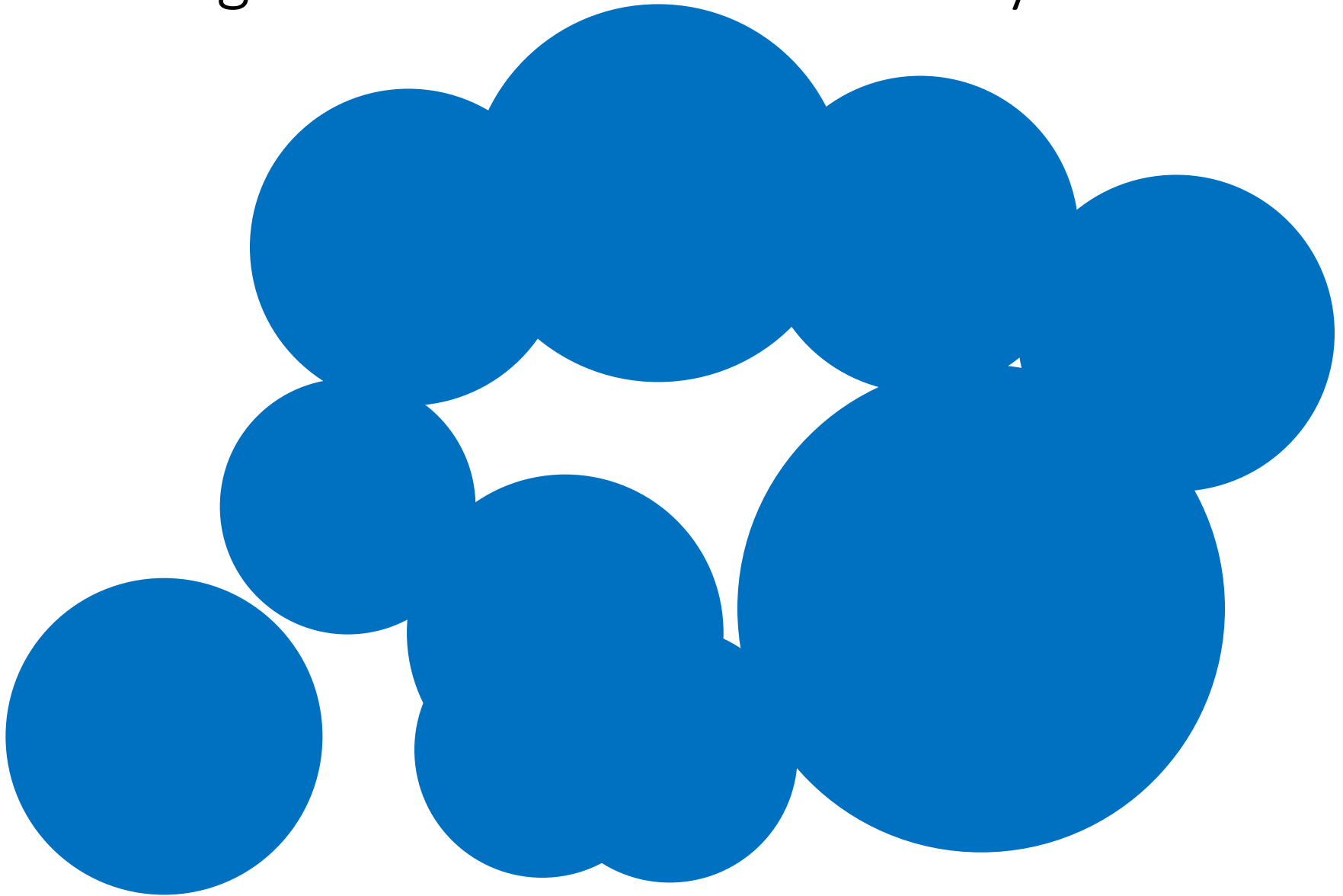
Arrangement of circles: how complex?



Arrangement of circles: TMI. Why?



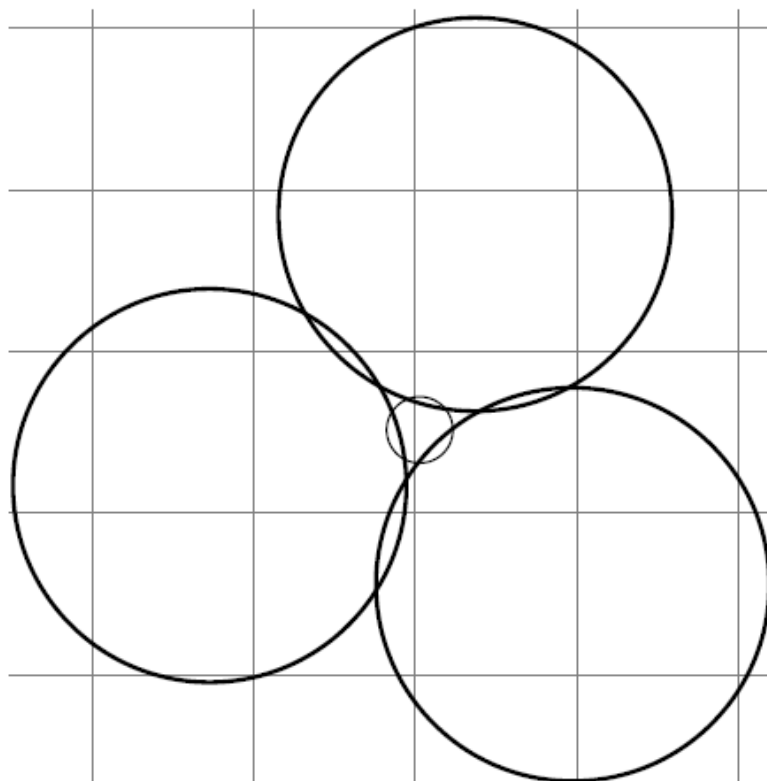
Arrangement of circles: TMI. Why?



Combinatorial analysis

- n – the number of obstacle discs
- arrangement of n circles
- the union of n discs
 - the lifting transform
 - the complexity of a 3-polytope

Combinatorial analysis, lower bound



Algorithms for computing the union of discs

- representation: DCEL
- Algorithm I: divide and conquer using plane sweep in the merge step
- Algorithm II: mimicking the proof of the combinatorial bound

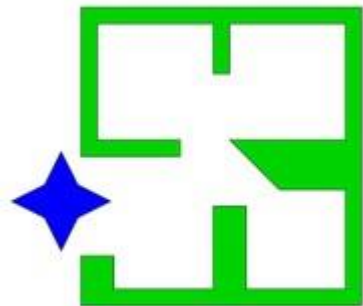
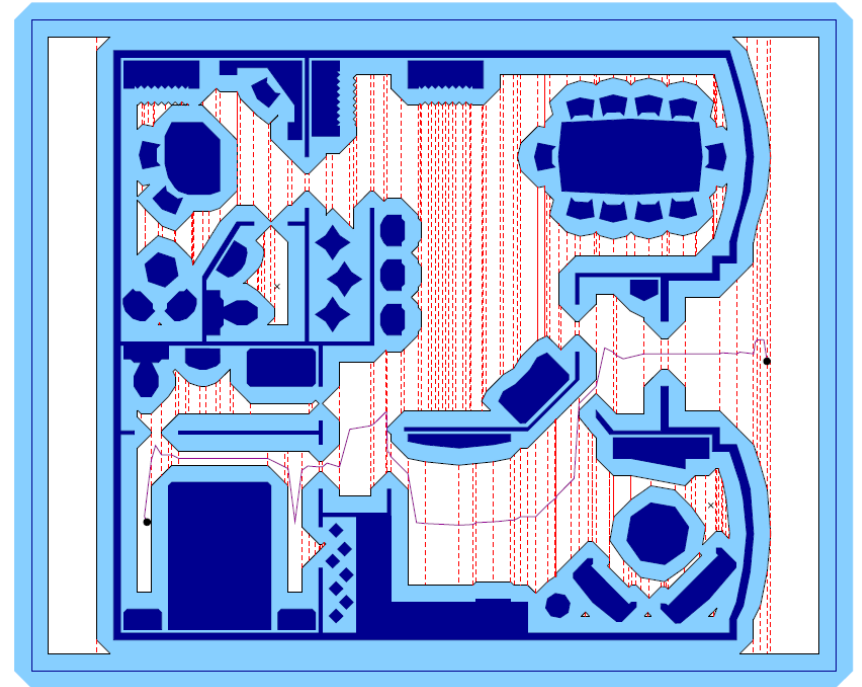
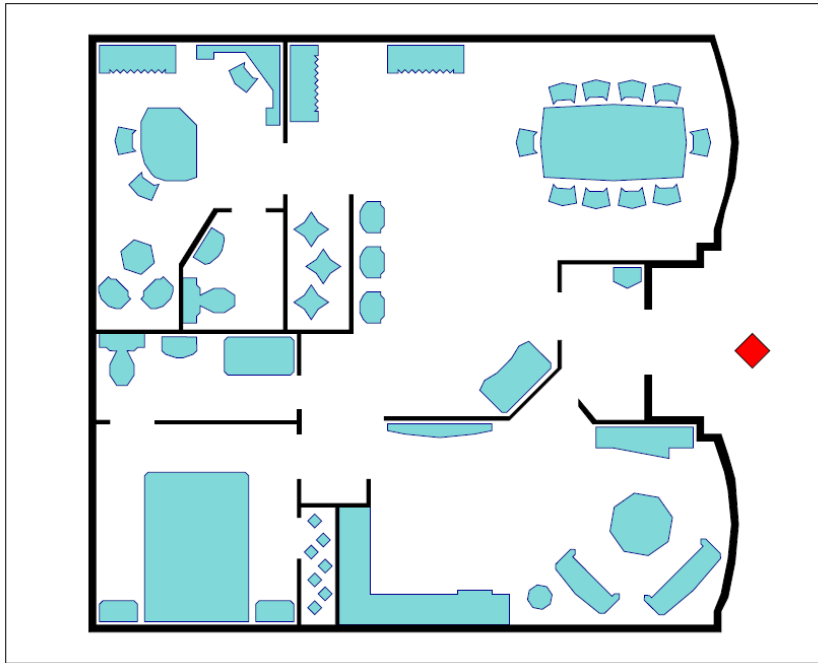
Algorithms for solving the Roomba MP problem

- augment the DCEL with vertical decomposition
- build a connectivity graph (CG) over the augmented DCEL:
 - a node for every free trapezoid
 - an edge between two trapezoids that share a vertical all
- find the cells that contain the start and goal positions
- search in the CG for a path between the nodes corresponding to the cells of the previous stage
- transform the path in the graph into a collision-free path in the plane

Reference

- Writeup on the course's website

The next step



THE END