Assignment no. 3

due: Sunday, May 8th, 2011

All the exercises in this assignment refer to the setting of a line segment (rod, ladder) translating and rotating among (not necessarily convex) polygonal obstacles in the plane.

Exercise 3.1 In class we saw a representation of the free space for the rod-among-polygons problem, the complexity of which is $O(n^5)$. The actual complexity of the free space in this motion-planning instance is much lower. **(a)** Show that the maximum combinatorial complexity of the free configuration space in this case is $O(n^2)$. To show this bound you have to bound the number of semi-free triple contacts (namely placements of the rod where it touches the obstacles boundaries in three points without penetrating into the obstacles). A hint will appear in the course's website later. **(b)** Show that the above bound is tight in the worst case. That is, describe a scene where the complexity of the free space is $\Omega(n^2)$.

Exercise 3.2 (p2) Devise and implement a simple and effective data structure for collision detection between a fixed length rod and a set of polygons in the plane. The set of polygons is fixed and a query specifies the position and orientation of the rod.

Exercise 3.3 (p2) Solve the rod-among-polygons motion planning problem with PRM, using your solution of Exercise 3.2 for collision detection.
(a) Write a brief description of your planner and its major ingredients.
(b) Present experimental results obtained with your program, depicting instances of varying difficulty for your planner, from easy to hard.
(c) Try two different distance measures (of your choice/design) between a pair of configurations. Design, carry out, and report on experiments that show the effect of the choice of distance measure on the performance of the planner.

Exercise 3.4 (optional, bonus) Give an efficient algorithm to solve the rod-among-polygons problem, and analyze its complexity.