Assignment no. 4

due: Monday, May 21st, 2018

Exercise 4.1 This exercise is concerned with simple proofs for probabilistic completeness and asymptotic near-optimality of PRM with a shrinking connection radius r_n. In particular, set $r_n = \gamma \left(\frac{\log n}{n}\right)^{1/d}$, where $\gamma = 2/\theta_d^{1/d}$, and θ_d is the volume of a d-dimensional unit hypersphere. Assume that the configuration space is $[0,1]^d$.
(a) Prove that PRM with the above r_n is probabilistically complete (of course, you cannot rely in your answer on the stronger results described in Lecture 6).
(b) Let $(\mathcal{F}, s, t)$ be a robust problem, and denote by c_ℓ^* the robust optimum with respect to the path length cost from s to t in the free space $\mathcal{F}$. Prove that for any fixed $\varepsilon > 0$, PRM returns a solution of length at most $(1+\varepsilon)\cdot 3 \cdot c_\ell^*$, with probability $1 - o(1)$.
optional, bonus: (c) Can you do better than the trivial stretch bound obtained in part (b)?

Exercise 4.2 In this exercise we will extend the proofs of asymptotic optimality for the integral cost function. Let r_n^* be the critical radius for continuum percolation. In class we saw that for $r_n > r_n^*$ PRM is (asymptotically) $(1+\varepsilon)\xi$-optimal with respect to path-length cost c_ℓ, and $(1+\varepsilon)$-optimal with respect to bottleneck cost c_b, for any fixed $\varepsilon > 0$.

Let $(\mathcal{F}, s, t)$ be a motion planning problem, and let $\mathcal{M} : \mathcal{F} \to \mathbb{R}$ be a cost map. The integral cost c_i of a path $\pi : [0,1] \to \mathcal{F}$ is defined to be

$$c_i(\pi) = \int_0^1 \mathcal{M}\left(\pi(\tau)\right) d\tau.$$

(a) State the necessary assumptions on $\mathcal{M}$ in order to prove asymptotic near-optimality of PRM, and provide a definition of the robust optimum c_i^*, with respect to c_i.
(b) Prove that for any fixed $\varepsilon > 0$ PRM obtains a solution of integral cost at most $(1+\varepsilon)\cdot\xi\cdot c_i^*$, with probability $1 - O(n^{-1})$.

Exercise 4.3 (p2) In this exercise we will improve the efficiency of your PRM implementation for a rod robot from Exercise 3.3 using an A*-driven construction of PRM. In all the experiments below use the connection radius $r_n = \frac{3}{2}\left(\frac{\log n}{n}\right)^{1/3}$.
(a) Implement an A*-based exploration of an implicit PRM, as we described in Lecture 7: The search begins with a set of vertices; whenever a vertex is expanded by A*, its neighbor set in the PRM graph is extracted by a NN-search method, and the corresponding edges are constructed (if necessary). You can use the lazy-evaluation or standard version of A* in your implementation.
(b) Design two admissible heuristics for the rod robot, based on one of the cost functions from Exercise 3.3.
(c) Present experimental results of your new planner. Compare the performance of your two heuristics (including path cost), as well as with the trivial heuristic, which returns 0 for any given input. Additionally, report whether you achieved any improvements in performance over your previous implementation.