Algorithmic Robotics and Motion Planning

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Motion planning and arrangements I: General considerations

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Overview

- Arrangements, reminder
- Arrangements and configuration spaces
- Examples
- General exact algorithms for motion planning
Reminder
What are arrangements?

Example: an arrangement of lines

vertex
edge
face
What are arrangements, cont’d

• an arrangement of a set S of geometric objects is the subdivision of space where the objects reside induced by S
• possibly non-linear objects (parabolas), bounded objects (segments, circles), higher dimensional (planes, simplices)
• numerous applications in robotics, molecular biology, vision, graphics, CAD/CAM, statistics, GIS
• have been studied for decades, originally mostly combinatorics nowadays mainly studied in combinatorial and computational geometry
Arrangements of lines: Combinatorics

The complexity of an arrangement is the overall number of cells of all dimensions comprising the arrangement. For planar arrangements, we count: vertices, edges, and faces.

The general position assumption: two lines meet in a single point, three lines have no point in common.
In an arrangement of \( n \) lines

number of vertices: \( n(n - 1)/2 \)

number of edges: \( n^2 \)

number of faces:

using Euler’s formula \(|V| - |E| + |F| = 2\)
we get \( n^2 + n^2 / 2 + 1 \)
Basic theorem of arrangement complexity

the maximum combinatorial complexity of an arrangement of $n$ well-behaved curves in the plane is $O(n^2)$; there are such arrangements whose complexity is $\Omega(n^2)$

more generally

the maximum combinatorial complexity of an arrangement of $n$ well-behaved (hyper)surfaces in $\mathbb{R}^d$ for a fixed $d$ is $O(n^d)$; there are such arrangements whose complexity is $\Omega(n^d)$
Configuration spaces

• arrangements $\mathcal{A}(\mathcal{S})$ are used for exact discretization of continuous problems

• a point $p$ in configuration space $\mathcal{C}$ has a property $\Pi(p)$

• if a neighborhood $U$ of $p$ is not intersected by an object in $\mathcal{S}$, the same property $\Pi(q)$ holds for every point $q \in U$ (the same holds when we restrict the configuration space to an object in $\mathcal{S}$)

• the objects in $\mathcal{S}$ are critical

• the property is invariant in each cell of the arrangement
Configuration space for translational motion planning

the rod is translating in the room

• the reference point: the lower end-point of the rod
• the configuration space is 2 dimensional
Configuration space obstacles

the robot has shrunk to a point

⇒

the obstacles are accordingly expanded
Critical curves in configuration space

the locus of *semi-free* placements
Making the connection:
The arrangement of critical curves in configuration space
Solving a motion-planning problem

a general framework

• what are the critical curves
• how complex is the arrangement of the critical curves
• constructing the arrangement and filtering out the forbidden cells
• what is the complexity of the free space
• can we compute the free space efficiently
• do we need to compute the entire free space?
Example: a disc moving among discs

- the critical curves are circles
- how complex is the arrangement of the circles?
- what is the complexity of the free space?
- can we compute the free space efficiently?
- do we need to compute the entire free space? does it matter?
Example: an L-shaped robot moving among points

- what are the critical curves?
- how complex is the arrangement of the critical curves?
- what is the complexity of the free space?
- how to compute the free space efficiently?

- next, we let the L rotate as well
- what are the critical surfaces?
- how complex is the arrangement of the critical surfaces?
- what is the complexity of the free space?
Complete solutions, I

the Piano Movers series [Schwartz-Sharir 83],
cell decomposition: a doubly-exponential solution, $O((nd)^{3^k})$ expected time

assuming the robot complexity is constant,
$k$ is the number of degrees of freedom,
$n$ is the complexity of the obstacles and
d is the algebraic complexity of the problem
From Piano Movers I (rod) to Piano Movers II (general)

• Ingredient: Combinatorics and algebra
• In PM I we saw both of them
• The algebra aspect: resultants and more
• The combinatorial aspect: arrg of Δs, for example

Solution to general MOP with 2 DOFs using CGAL, following the separation of algebra and combinatorics (later)
Complete solutions, II

roadmap [Canny 87]:
a singly exponential solution,
\( n^k (\log n) d^{O(k^2)} \) expected time

see also [Basu-Pollack-Roy 06]
Bibliography

References to all the results mentioned in this presentation and more can be found in the following two chapters of the:


• Chapter 28, Arrangements, Halperin and Sharir
• Chapter 50, Algorithmic Motion Planning, Halperin-Slazman-Sharir
THE END