Assignment no. 3  
due: December 14th, 2015

Exercise 3.1 On $n$ parallel railway tracks $n$ trains are going with constant speeds $v_1, v_2, \ldots, v_n$. At time $t = 0$ the trains are at positions $k_1, k_2, \ldots, k_n$. Give an $O(n \log n)$ time algorithm that detects all trains that at some moment in time are leading.

Exercise 3.2 A simple polygon $P$ is called star-shaped if it contains a point $q$ such that for any point $p$ in $P$ the line segment $pq$ is contained in $P$. Give a randomized algorithm with expected linear running time to decide whether a simple polygon is star-shaped.

Exercise 3.3 Instead of removing the object from the mold by a single translation (as we saw in class), we can also try to remove it by a single rotation. For simplicity let’s consider the planar variant of this casting problem, and let’s only look at clockwise rotations.

(a) Give an example of a simple polygon $P$ with top facet $f$ that is not castable when we require that $P$ should be removed from the mold by a single translation, but that is castable using rotation around a point.

(b) Show that the problem of finding a center of rotation that allows us to remove $P$ with a single rotation from its mold can be reduced to the problem of finding a point in the common intersection of a set of half-planes.

(CGAA Ex. 4.7)

Exercise 3.4 Give an example of a set of $n$ points in the plane, and a query rectangle for which the number of nodes of the kd-tree visited is $\Omega(\sqrt{n})$.

Exercise 3.5 The algorithm we saw in class for searching in a kd-tree (where the search is guided by comparing the region of a node with the query region) can also be used when querying with ranges other than rectangles. For example, a query is answered correctly if the range is a triangle.

(a) Show that the query time for range queries with triangles is linear in the worst case, even if no points are reported at all. Hint: Choose all the input points to lie on the line $y = x$.

(b) Suppose that a data structure is needed that can answer triangular range queries but only for triangles whose edges are horizontal, vertical or have slope 1 or $-1$. Devise a linear size data structure that answers such queries in $O(n^{3/4} + k)$ time, where $k$ is the number of points to be reported. Hint: Choose 4 coordinate axes in the plane and use a “4-dimensional” kd-tree.

(c) Improve the query time to $O(n^{2/3} + k)$.