Computational Geometry — Fall 2016/17 — Dan Halperin

Assignment no. 3

due: January 2nd, 2017

Exercise 3.1 On n parallel railway tracks n trains are going with constant speeds $v_1, v_2, \ldots, v_n$. At time $t = 0$ the trains are at positions $k_1, k_2, \ldots, k_n$. Give an $O(n \log n)$ time algorithm that detects all trains that at some moment in time are leading.

Exercise 3.2 A simple polygon P is called *star-shaped* if it contains a point q such that for any point p in P the line segment $\overline{pq}$ is contained in P. Give a randomized algorithm with expected linear running time to decide whether a simple polygon is star-shaped.

Exercise 3.3 Instead of removing the object from the mold by a single translation (as we saw in class), we can also try to remove it by a single rotation. For simplicity let's consider the planar variant of this casting problem, and let's only look at clockwise rotations.
(a) Give an example of a simple polygon P with top facet f that is not castable when we require that P should be removed from the mold by a single translation, but that is castable using rotation around a point.
(b) Show that the problem of finding a center of rotation that allows us to remove P with a single rotation from its mold can be reduced to the problem of finding a point in the common intersection of a set of half-planes.
(CGAA Ex. 4.7)

Exercise 3.4, bonus Consider the restricted version of the casting problem in which we insist that the object is removed from its mold using a vertical translation (perpendicular to the top facet).
(a) Prove that in this case there is always only a constant number of possible top facets.
(b) Give a linear time algorithm that determines whether for a given object a mold exists under this restricted model.