# Assignment no. 3

due: December 25th, 2018

**Exercise 3.1** On $n$ parallel railway tracks $n$ trains are going with constant speeds $v_1, v_2, \ldots, v_n$. At time $t = 0$ the trains are at positions $k_1, k_2, \ldots, k_n$. Give an $O(n \log n)$ time algorithm that detects all trains that at some moment in time are leading.

**Exercise 3.2** A simple polygon $P$ is called *star-shaped* if it contains a point $q$ such that for any point $p$ in $P$ the line segment $\overline{pq}$ is contained in $P$. Give a randomized algorithm with expected linear running time to decide whether a simple polygon is star-shaped.

**Exercise 3.3** Instead of removing the object from the mold by a single translation (as we saw in class), we can also try to remove it by a single rotation. For simplicity let's consider the planar variant of this casting problem, and let's only look at clockwise rotations.
(a) Give an example of a simple polygon $P$ with top facet $f$ that is not castable when we require that $P$ should be removed from the mold by a single translation, but that is castable using rotation around a point.
(b) Show that the problem of finding a center of rotation that allows us to remove $P$ with a single rotation from its mold can be reduced to the problem of finding a point in the common intersection of a set of half-planes.
(CGAA Ex. 4.7)

**Exercise 3.4** Describe in detail the procedure **UnboundedLP3**, whose input is an objective function and a set of half-spaces in $\mathbb{R}^3$. The procedure decided whether the underlying linear program is unbounded. If it is, the procedure computes a ray in the feasible region such that the objective improves as we proceed along the ray away from its terminus. If the program is bounded, the procedure returns three of the input half-spaces as witnesses to this effect. Analyze the running time of the procedure.
No need to prove the correctness of the procedure.
No need to provide the porcedure for the bounded case!