

Assignment no. 2

due: December 4th, 2017

Exercise 2.1 Let S be a set of n disjoint line segments in the plane, and let p be a point not on any of the segments in S . We wish to determine all the line segments of S that p can see, namely, all the line segments of S that contain some point q so that the open segment \overline{pq} does not intersect any line segment of S . Give an $O(n \log n)$ time algorithm to solve this problem. (CGAA Ex. 2.14)

Exercise 2.2 The *stabbing number* of a triangulation of a simple polygon P is the maximum number of diagonals intersected by any line segment interior to P . Give an algorithm that computes a triangulation of a convex polygon that has stabbing number $O(\log n)$.

Exercise 2.3 Prove that the following polyhedron \mathcal{P} cannot be tetrahedralized using only vertices of \mathcal{P} , namely its interior cannot be partitioned into tetrahedra whose vertices are selected from the vertices of \mathcal{P} (see the enclosed figure).¹

Let a, b, c be the vertices (labeled counterclockwise) of an equilateral triangle in the xy -plane. Let a', b', c' be the vertices of abc when translated up to the plane $z = 1$. Define an intermediate polyhedron \mathcal{P}' as the hull of the two triangles including the diagonal edges ab', bc' , and ca' , as well as the vertical edges aa', bb' , and cc' , and the edges of the two triangles abc and $a'b'c'$. Now twist the top triangle $a'b'c'$ by 30° in the plane $z = 1$, rotating and stretching the attached edges accordingly: this is the polyhedron \mathcal{P} .

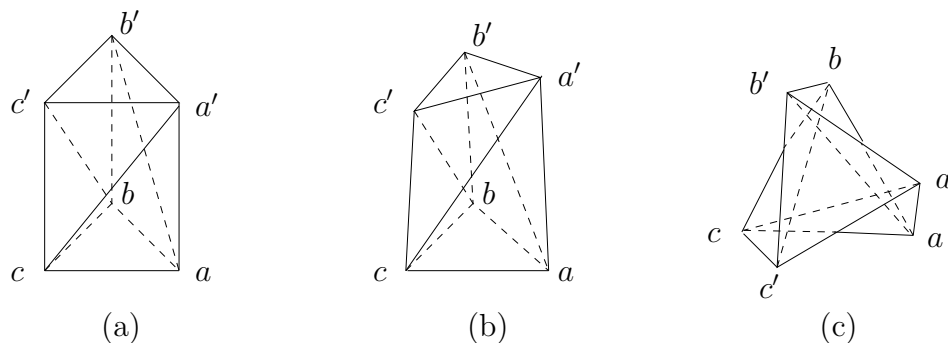


Figure 1: The untetrahedralizable polyhedron is constructed by twisting the top of a triangular prism (a) by 30° degrees, producing (b), shown in top view in (c)

Notice that there are additional exercises on the other side of the page.

¹This construction is due to Schönhardt, 1928. The description here is taken from O'Rourke's *Art Gallery Theorems and Algorithms*.

Exercise 2.4 The *pockets* of a simple polygon are the areas outside the polygon, but inside its convex hull. Let P_1 be a simple polygon with n_1 vertices, and assume that a triangulation of P_1 as well as of its pockets is given. Let P_2 be a convex polygon with n_2 vertices. Show that the intersection $P_1 \cap P_2$ can be computed in $O(n_1 + n_2)$ time. (CGAA Ex. 3.12)

Remarks. (1) The fine details matter—provide a precise description of your solution. (2) You may assume that the triangulation of P_1 together with the triangulation of its pockets, is given as a DCEL. (3) It is helpful to partition the algorithm into cases based on the relative placement of P_2 and the convex hull of P_1 : are they disjoint, contained in one another or partly intersecting.

Exercise 2.5 In each of the following settings, give an example where the specified **vertex guards** do not fully cover the polygon in the art gallery sense:

(a) A simple polygon with $2k$ vertices, for every $k > 2$, and a specific assignment of guards placed at **every other vertex** along the boundary of the polygon. Namely, guards placed at the vertices $v_i, v_{i+2}, v_{i+4}, \dots$, do not fully cover the polygon.

(b) Similarly, a simple polygon with $3k$ vertices, for every $k > 2$, and a specific assignment of guards placed at **every third vertex** along the boundary of the polygon.

(c) A simple polygon with n vertices, for every $n > 5$, and guards placed only at **convex vertices**. A vertex is convex if its interior angle is less than π .

(d, **bonus**) A simple polygon with $3n$ vertices, and any assignment of guards placed at **every third vertex** along the boundary of the polygon. Notice the difference between this item and Item (b): (i) Here you need to find just one polygon, for one value of n , and (ii) unlike Item (b) where you need to find just one specific assignment per polygon, here every assignment of guards to every third vertex, no matter what is your starting vertex, should not cover the polygon.