Computational Geometry — Fall 2017/18 — Dan Halperin

Assignment no. 5

Not for submission

Exercise 5.1 Let P be a set of points in the plane. Let $V(p)$ denote the Voronoi cell of a point site $p \in P$. We construct the dual graph G of the Voronoi diagram as follows: every node in G corresponds to a Voronoi cell, and two nodes in G have an arc between them if the corresponding cells share an edge. Consider a straight-line embedding of G where the node corresponding to $V(p)$ is the point p, and the arc connecting the nodes of $V(p)$ and $V(q)$ is the line segment $\overline{pq}$. We call this embedding the Daelaunay graph of P. Prove that the Delaunay graph of a planar point set is a plane graph.

Exercise 5.2 Let P be a set of points in the plane in general position, in particular no four points of P lie on a circle. Show that a triangualtion T of P is legal if an only if T is the Delaunay triangulation of P.

Exercise 5.3 To complete the sweep-line algorithm for computing the Voronoi diagram of points in the plane, write a procedure to compute a sufficiently large bounding box from the incomplete doubly-connected edge list and the tree T (the status structure) after the sweep is finished. The box should contain all sites and all Voronoi vertices.

Exercise 5.4 We define the *level* of a point in an arrangement of lines in the plane to be the number of lines strictly above it. Given a set L of n lines in the plane, give an $O(n \log n)$ time algorithm to compute the maximum level of any vertex in the arrangement $A(L)$.

Exercise 5.5 Let S be a set of n segments in the plane. We want to preprocess S into a data structure that can answer the following query: Given a query line ℓ, how many segments in S does it intersect?
(a) Formulate the problem in the dual plane.
(b) Describe a data structure for this problem that uses $O(n^2)$ expected storage and has $O(\log n)$ expected query time.
(c) Describe how the data structure can be built in $O(n^2 \log n)$ expected time.