Computational Geometry — Spring 2010 — Dan Halperin

Assignment no. 2

due: April 12th, 2010

Please submit each exercise on a separate sheet (or sheets), with your name and student id number on it.

Exercise 2.1 The *pockets* of a simple polygon are the areas outside the polygon, but inside its convex hull. Let P_1 be a simple polygon with n_1 vertices, and assume that a triangulation of P_1 as well as of its pockets is given. Let P_2 be a convex polygon with n_2 vertices. Show that the intersection $P_1 \cap P_2$ can be computed in $O(n_1 + n_2)$ time. (CGAA Ex. 3.12)

Exercise 2.2 The *stabbing number* of a triangulation of a simple polygon P is the maximum number of diagonals intersected by any line segment interior to P. Give an algorithm that computes a triangulation of a convex polygon that has stabbing number $O(\log n)$.

Exercise 2.3 Prove that the following polyhedron $\mathcal{P}$ cannot be tetrahedralized using only vertices of $\mathcal{P}$, namely its interior cannot be partitioned into tetrahedra whose vertices are selected from the vertices of $\mathcal{P}$ (see the enclosed figure).[1]

> Let a, b, c be the vertices (labeled counterclockwise) of an equilateral triangle in the xy-plane. Let a', b', c' be the vertices of abc when translated up to the plane $z = 1$. Define an intermediate polyhedron $\mathcal{P}'$ as the hull of the two triangles including the diagonal edges ab', bc', and ca', as well as the vertical edges aa', bb', and cc', and the edges of the two triangles abc and $a'b'c'$. Now twist the top triangle $a'b'c'$ by 30° in the plane $z = 1$, rotating and stretching the attached edges accordingly: this is the polyhedron $\mathcal{P}$.

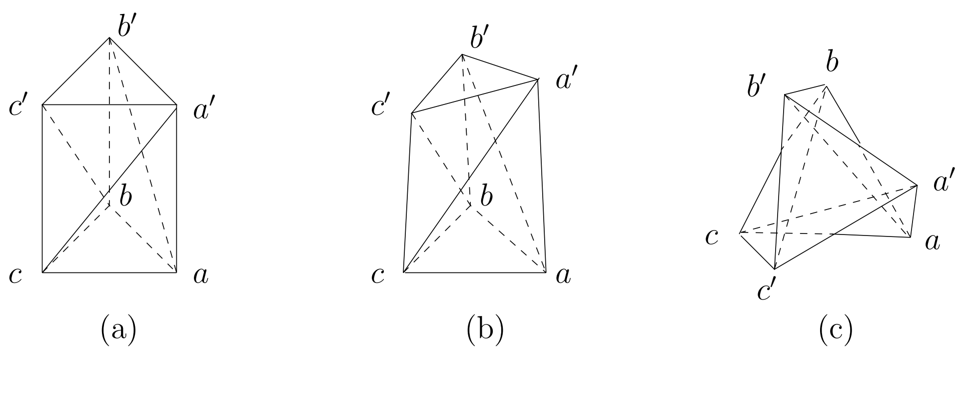

Figure 1: The untetrahedralizable polyhedron is constructed by twisting the top of a triangular prism (a) by 30° degrees, producing (b), shown in top view in (c)

Notice that there is an additional exercise on the other side of the page.

[1] This construction is due to Schönhardt, 1928. The description here is taken from O'Rourke's *Art Gallery Theorems and Algorithms.*

Exercise 2.4 Let P be a simple polygon with n vertices and let G be a set of k points inside P, which are the placements of cameras. Give an algorithm to determine whether the cameras in G cover the polygon P (in the "art-gallery" sense). Analyze the complexity of your algorithm.