Computational Geometry — Spring 2010 — Dan Halperin

Assignment no. 3
due: May 24th, 2010

Please submit each exercise on a separate sheet (or sheets), with your name and student id number on it.

Exercise 3.0 Study the deterministic linear time algorithm for solving two-variable linear programs by Meggido. It is clearly described in Section 7.2.5, TWO-VARIABLE LINEAR PROGRAMMING of the Computational Geometry book by Preparata and Shamos, the 1985 Edition. **No need to submit anything here.**

Exercise 3.1 On n parallel railway tracks n trains are going with constant speeds $v_1, v_2, \ldots, v_n$. At time $t = 0$ the trains are at positions $k_1, k_2, \ldots, k_n$. Give an $O(n \log n)$ time algorithm that detects all trains that at some moment in time are leading.

Exercise 3.2 A simple polygon P is called *star-shaped* if it contains a point q such that for any point p in P the line segment $\overline{pq}$ is contained in P. Give a linear time algorithm to decide whether a simple polygon is star-shaped.

Exercise 3.3 Describe in detail the procedure UnboundedLP3, which accepts n halfspaces in three-dimensional space, and an objective function. The procedure either finds that the induced LP is infeasible, or that the LP is bounded in which case UnboundedLP3 outputs three witnesses to this fact, or outputs a ray (in three-dimensional space) such that as we proceed away from the ray's terminus, the objective function grows. Show that the procedure runs in time $O(n)$.

Exercise 3.4 Give an example of a set of n points in the plane, and a query rectangle for which the number of nodes of the kd-tree visited is $\Omega(\sqrt{n})$.

Exercise 3.5 The algorithm we saw in class for searching in a kd-tree (where the search is guided by comparing the *region* of a node with the query region) can also be used when querying with ranges other than rectangles. For example, a query is answered correctly if the range is a triangle.
(a) Show that the query time for range queries with triangles is linear in the worst case, even if no points are reported at all. Hint: Choose all the input points to lie on the line $y = x$.
(b) Suppose that a data structure is needed that can answer triangular range queries but only for triangles whose edges are horizontal, vertical or have slope $+1$ or -1. Devise a linear size data structure that answers such queries in $O(n^{3/4} + k)$ time, where k is the number of points to be reported. Hint: Choose 4 coordinate axes in the plane and use a "4-dimensional" kd-tree.
(c) Improve the query time to $O(n^{2/3} + k)$.