

### Assignment no. 2

due: April 7th, 2014

**Exercise 2.1** The *pockets* of a simple polygon are the areas outside the polygon, but inside its convex hull. Let  $P_1$  be a simple polygon with  $n_1$  vertices, and assume that a triangulation of  $P_1$  as well as of its pockets is given. Let  $P_2$  be a convex polygon with  $n_2$  vertices. Show that the intersection  $P_1 \cap P_2$  can be computed in  $O(n_1 + n_2)$  time. (CGAA Ex. 3.12)

**Exercise 2.2** The *stabbing number* of a triangulation of a simple polygon  $P$  is the maximum number of diagonals intersected by any line segment interior to  $P$ . Give an algorithm that computes a triangulation of a convex polygon that has stabbing number  $O(\log n)$ .

**Exercise 2.3** Prove that the following polyhedron  $\mathcal{P}$  cannot be tetrahedralized using only vertices of  $\mathcal{P}$ , namely its interior cannot be partitioned into tetrahedra whose vertices are selected from the vertices of  $\mathcal{P}$  (see the enclosed figure).<sup>1</sup>

Let  $a, b, c$  be the vertices (labeled counterclockwise) of an equilateral triangle in the  $xy$ -plane. Let  $a', b', c'$  be the vertices of  $abc$  when translated up to the plane  $z = 1$ . Define an intermediate polyhedron  $\mathcal{P}'$  as the hull of the two triangles including the diagonal edges  $ab', bc'$ , and  $ca'$ , as well as the vertical edges  $aa', bb'$ , and  $cc'$ , and the edges of the two triangles  $abc$  and  $a'b'c'$ . Now twist the top triangle  $a'b'c'$  by  $30^\circ$  in the plane  $z = 1$ , rotating and stretching the attached edges accordingly: this is the polyhedron  $\mathcal{P}$ .

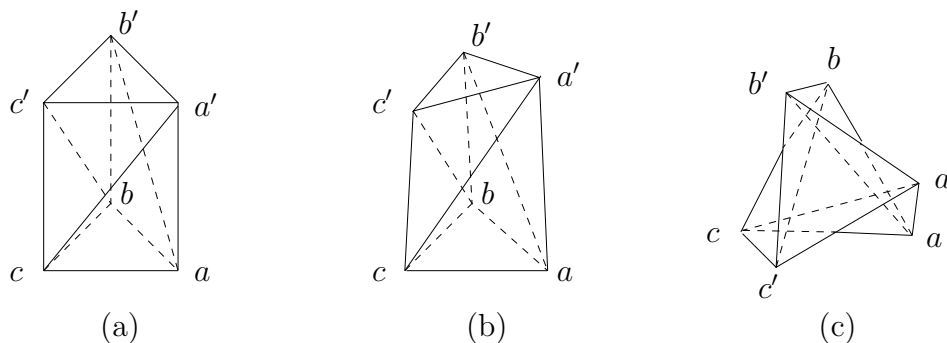


Figure 1: The untetrahedralizable polyhedron is constructed by twisting the top of a triangular prism (a) by  $30^\circ$  degrees, producing (b), shown in top view in (c)

Notice that there are additional exercises on the other side of the page.

<sup>1</sup>This construction is due to Schönhardt, 1928. The description here is taken from O'Rourke's *Art Gallery Theorems and Algorithms*.

**Exercise 2.4 (revised, 26.3.14)** Can you come up with examples of polygons such that the following sets of vertex guards do not fully cover the polygon in the art gallery sense:

(a) A simple polygon with  $2k$  vertices, for every  $k > 2$ , and a specific assignment of guards placed at every other vertex along the boundary of the polygon.

(b) A simple polygon with  $n$  vertices, for every  $n > 5$ , and guards placed only at convex vertices.

**Exercise 2.5** On  $n$  parallel railway tracks  $n$  trains are going with constant speeds  $v_1, v_2, \dots, v_n$ . At time  $t = 0$  the trains are at positions  $k_1, k_2, \dots, k_n$ . Give an  $O(n \log n)$  time algorithm that detects all trains that at some moment in time are leading.

**Exercise 2.6** A simple polygon  $P$  is called *star-shaped* if it contains a point  $q$  such that for any point  $p$  in  $P$  the line segment  $\overline{pq}$  is contained in  $P$ . Give a linear expected-time algorithm to decide whether a simple polygon is star-shaped.