Assignment no. 1

due: March 25th, 2019

Many exercises (but not all) will be taken from the main textbook of the course *Computational Geometry Algorithms and Applications*[1] (CGAA). Some exercises are similar but not identical to exercises in the book. When a figure in the book could be helpful, the exercise number in the book is given.

Exercise 1.1 In many situations we need to compute convex hulls of objects other than points.
(a) Let T be a set of n triangles in the plane. Prove that the convex hull of T is exactly the same as the convex hull of the $3n$ endpoints of the triangles.
(b) Devise an algorithm to compute the convex hull of a set of m possibly intersecting convex polygons in the plane, with a total of n vertices. Let h denote the number of vertices on the boundary of the desired convex hull. The algorithm should run in $O(mh+n)$ time—show that this is indeed the running time of your algorithm. Analyze the storage requirement of the algorithm.

Exercise 1.2 Describe in detail a "gift-wrapping" algorithm for computing the convex hull of a finite set of points in three-dimensional space and analyze its running time. You may assume that the input points are in general position, which means, in particular, that no four points lie on a common plane, that no three points lie on a common line, etc.

Exercise 1.3 Let R be a set of n disjoint triangles in the plane. We want to find a set of $n-1$ segments with the following properties:

- Each segment connects a point on the boundary of one triangle to a point on the boundary of another triangle.
- The interiors of segments are pairwise disjoint and they are disjoint from the triangles
- Together they connect all the triangles to each other, that is, by walking along the segments and the triangle boundaries it must be possible to walk from a triangle to any other triangle.

Develop a plane sweep algorithm for this problem that runs in $O(n \log n)$ time. State the events and the data structures that you use explicitly, and describe the cases that arise and the actions required for each of them. Also state the sweep invariant. (CGAA Ex. 2.13)

Exercise 1.4 Let S be a set of n disjoint line segments in the plane, and let p be a point not on any of the segments in S. We wish to determine all the line segments of S that p can see, namely, all the line segments of S that contain some point q so that the open segment $\overline{pq}$ does not intersect any line segment of S. Give an $O(n \log n)$ time algorithm to solve this problem. (CGAA Ex. 2.14)

[1] M. de Berg, M. van Kreveld, M. Overmars, and O. Schwarzkopf, *Computational Geometry: Algorithms and Applications*, 3rd Edition, Springer, 2008.