Assignment no. 2

due: April 8th, 2019

Exercise 2.1 Let S be a planar subdivision of complexity n, and let P be a set of m points. Give a plane-sweep algorithm that computes for every point in P in which face of S it is contained. Show that your algorithm runs in $O((n+m)\log(n+m))$ time.

Exercise 2.2 Let L be a set of n lines in the plane. Give an $O(n \log n)$ time algorithm to compute an axis-parallel rectangle that contains all the vertices of the arrangement $\mathcal{A}(L)$ in its interior.

Exercise 2.3 Hopcroft's problem is to decide, given n lines and n points in the plane, whether any point is contained in any line. Give an $O(n^{3/2} \log n)$ time algorithm to solve Hopcroft's problem. Hint: Give an $O(n \log n)$ time algorithm to decide, given n lines and $\sqrt{n}$ points in the plane, whether any point is contained in any line.

Exercise 2.4 Prove that the following polyhedron $\mathcal{P}$ cannot be tetrahedralized using only vertices of $\mathcal{P}$, namely its interior cannot be partitioned into tetrahedra whose vertices are selected from the vertices of $\mathcal{P}$ (see the figure below).[1]

> Let a, b, c be the vertices (labeled counterclockwise) of an equilateral triangle in the xy-plane. Let a', b', c' be the vertices of abc when translated up to the plane $z = 1$. Define an intermediate polyhedron $\mathcal{P}'$ as the hull of the two triangles including the diagonal edges ab', bc', and ca', as well as the vertical edges aa', bb', and cc', and the edges of the two triangles abc and $a'b'c'$. Now twist the top triangle $a'b'c'$ by $30°$ in the plane $z = 1$, rotating and stretching the attached edges accordingly: this is the polyhedron $\mathcal{P}$.

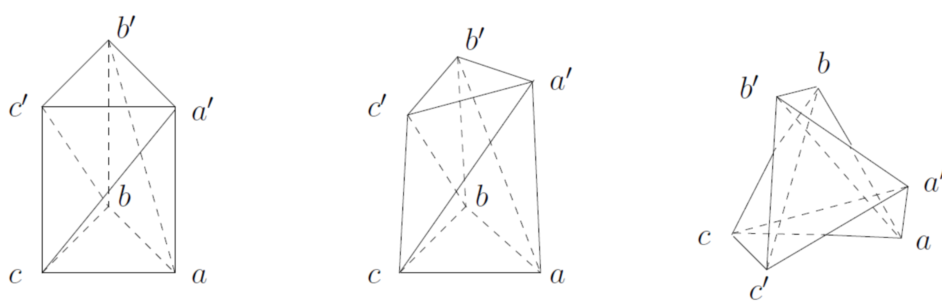

Figure 1: The untetrahedralizable polyhedron is constructed by twisting the top of a triangular prism (a) by 30° degrees, producing (b), shown in top view in (c)

[1] This construction is due to Schönhardt, 1928. The description here is taken from O'Rourke's *Art Gallery Theorems and Algorithms*.