Intro to sampling-based motion-planning and motion-planning via Manifold Samples*

Oren Salzman, Tel-Aviv University

*Joint work with Michael Hemmer, Barak Raveh and Dan Halperin

TEL AVIV UNIVERSITY

Outline

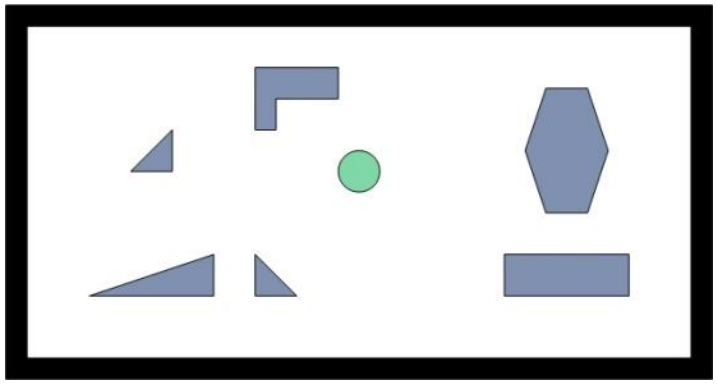
- Sampling-based motion-planning
- Motion Planning via Manifold Samples (MMS)
- MMS for 3DoF polygonal robot

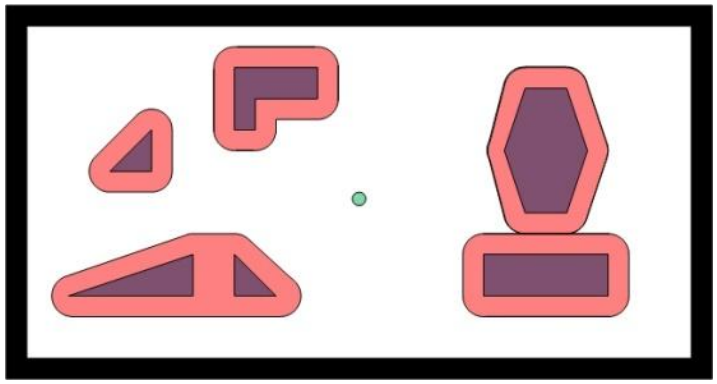
Outline

- Sampling-based motion-planning
- Motion Planning via Manifold Samples (MMS)
- MMS for 3DoF polygonal robot

Motion Planning - Definitions

- **Workspace** – A description of the (2D or 3D) world consisting of a **robot** and **obstacles**
- **Configuration Space- (C)** The space of parameters that define the robot's position and orientation in the workspace



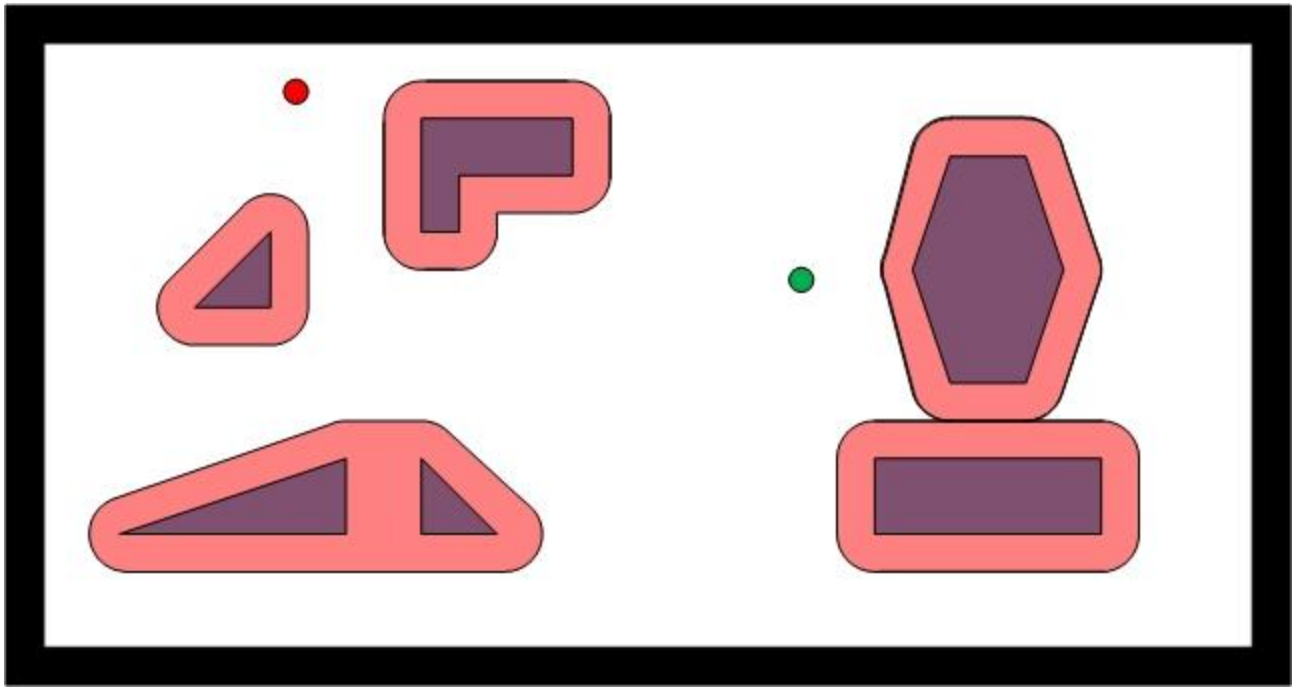


Motion Planning - Definitions

- **Workspace** – A description of the (2D or 3D) world consisting of a **robot** and **obstacles**
- **Configuration Space- (C)** The space of parameters that define the robot's position and orientation in the workspace
- **Degrees of Freedom-** The minimal number of parameters required to uniquely define a position of the robot
- **Free Space** (C_{free})- Set of collision-free configurations
- **Forbidden Space (C_{forb})-** $C \setminus C_{free}$

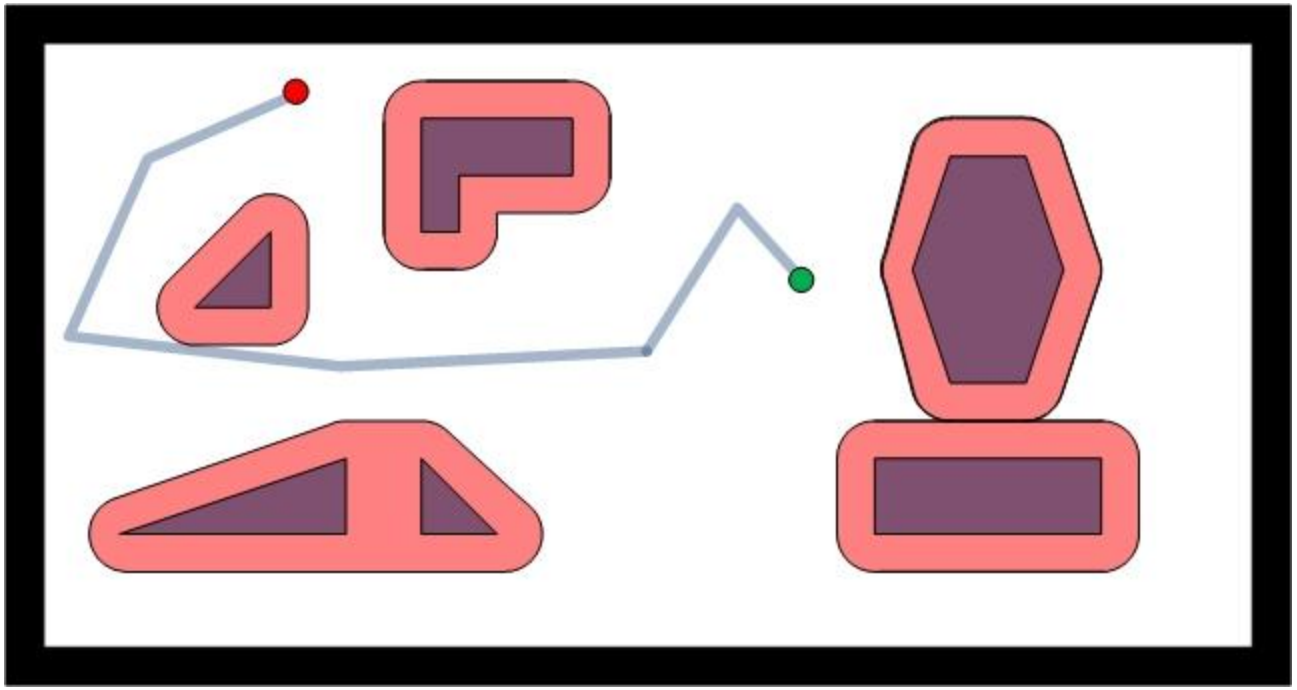
Motion Planning - Objective

- Find a path in C_{free} from a free source configuration to a free target configuration



Motion Planning - Objective

- Find a path in C_{free} from a free source configuration to a free target configuration



Algorithmic Approaches for Motion Planning

- Sampling-Based Planning
  - Capture connectivity of C_{free} by randomly **sampling** configurations
- Combinatorial Motion Planning
  - **Analytically** compute an explicit combinatorial representation of C_{free}

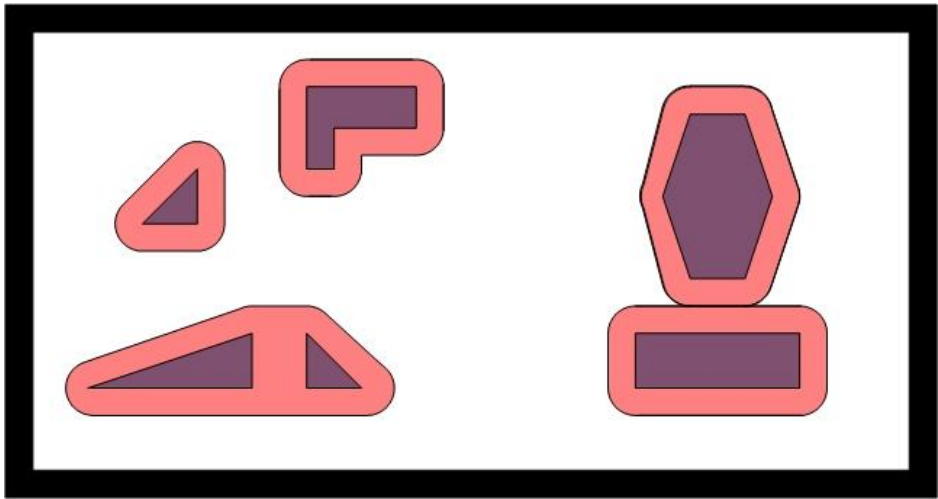
Algorithmic Approaches for Motion Planning

- Sampling-Based Planning
  - Capture connectivity of C_{free} by randomly **sampling** configurations

- Kavraki, Svestka, Latombe, Overmars 96: Probabilistic roadmaps for path planning in high dimensional configuration spaces (PRM)
- LaValle 98: Rapidly-exploring random trees: A new tool for path planning (RRT)
- Hsu, Latombe, Motwani 99: Path planning in expansive configuration spaces (EST)

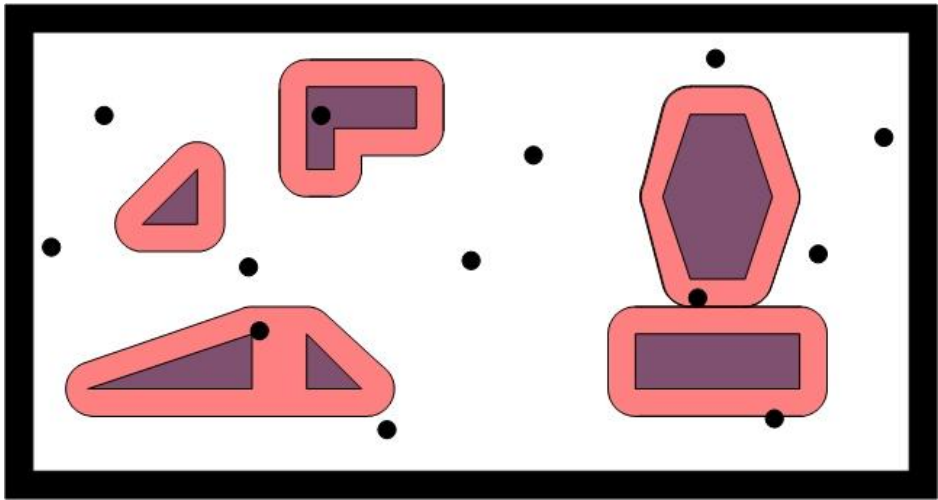
Sampling-Based Planning - PRM

- Multi query planner
- Preprocesses configuration space into a graph (roadmap)



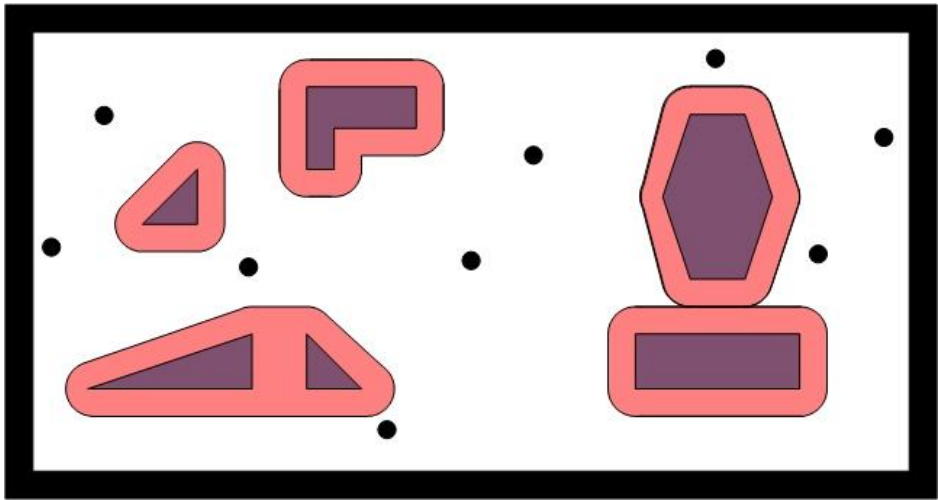
Sampling-Based Planning - PRM

- Multi query planner
- Preprocesses configuration space into a graph (roadmap)
  - Randomly sample n robot configurations



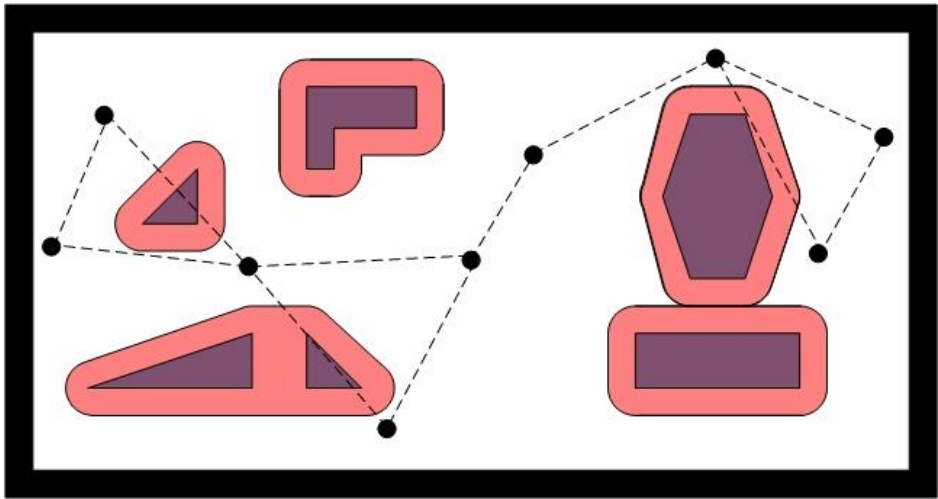
Sampling-Based Planning - PRM

- Multi query planner
- Preprocesses configuration space into a graph (roadmap)
  - Randomly sample n robot configurations
  - Discard invalid configurations (collision detection)



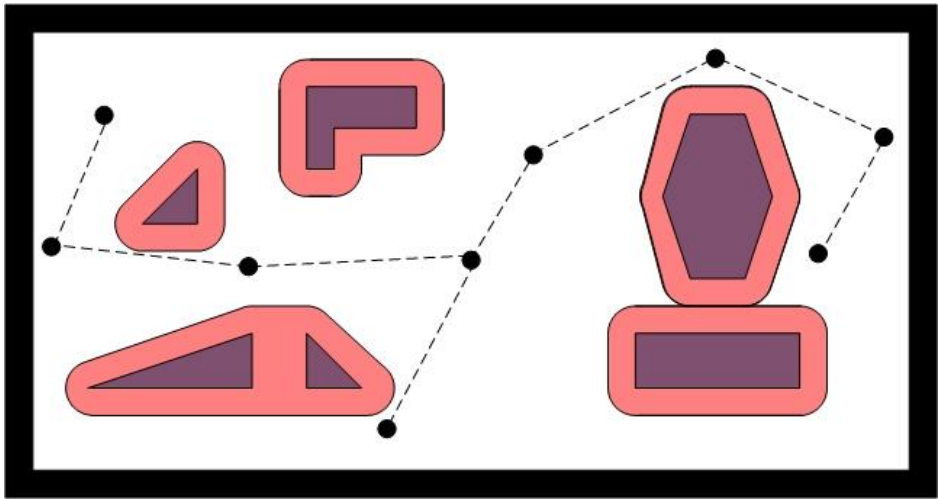
Sampling-Based Planning - PRM

- Multi query planner
- Preprocesses configuration space into a graph (roadmap)
  - Randomly sample n robot configurations
  - Discard invalid configurations (collision detection)
  - Connect close-by configurations by dense sampling



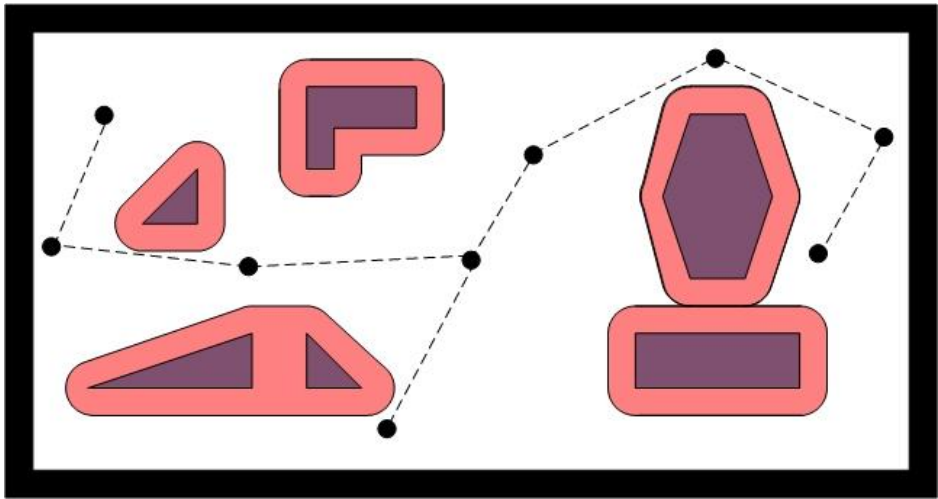
Sampling-Based Planning - PRM

- Multi query planner
- Preprocesses configuration space into a graph (roadmap)
  - Randomly sample n robot configurations
  - Discard invalid configurations (collision detection)
  - Connect close-by configurations by dense sampling
  - Discard invalid edges (local-planning)



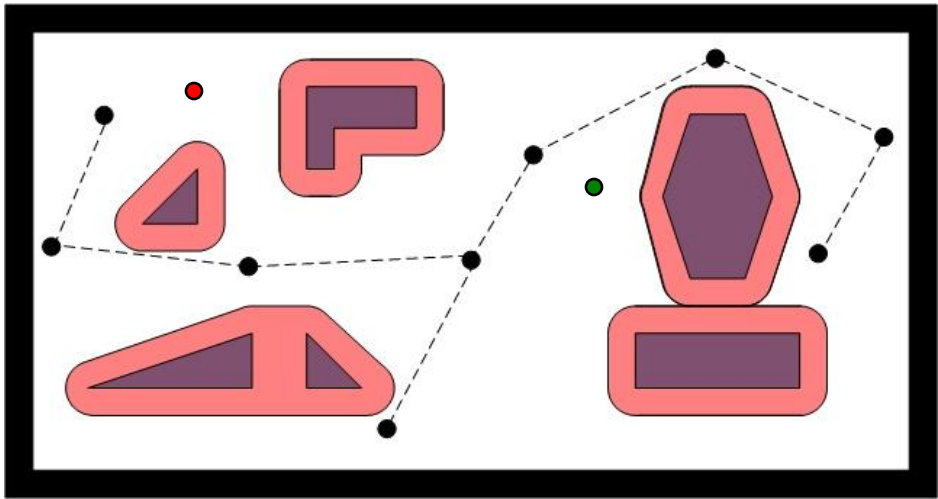
Sampling-Based Planning - PRM

- Query



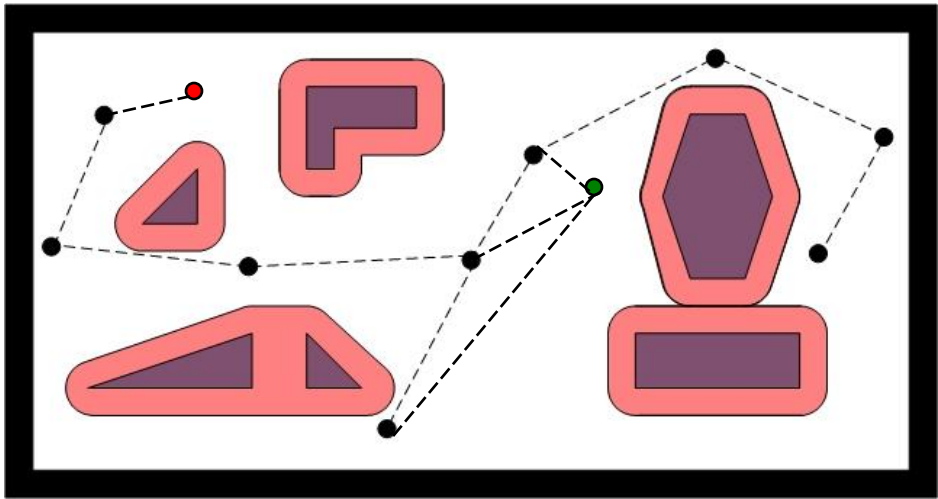
Sampling-Based Planning - PRM

- Query
  - Add source & target to roadmap



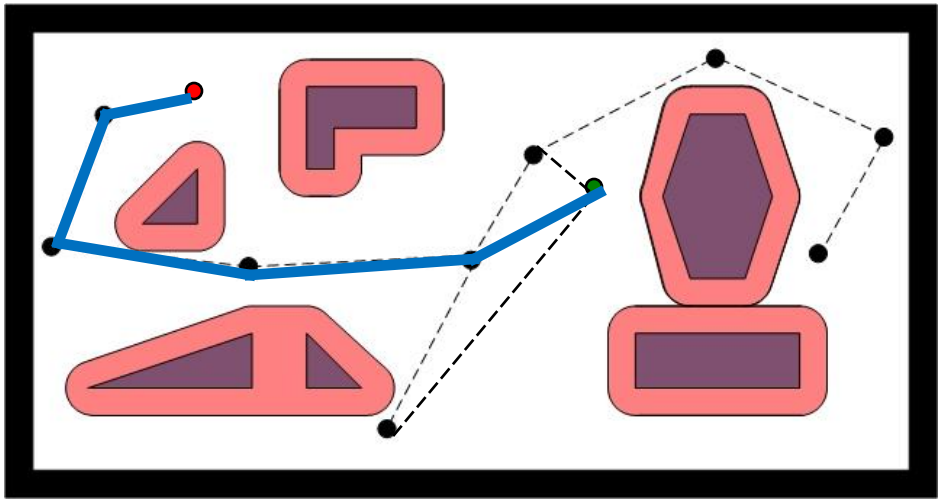
Sampling-Based Planning - PRM

- Query
  - Add source & target to roadmap
  - Add connections to nearest neighbors for source and target



Sampling-Based Planning - PRM

- Query
  - Add source & target to roadmap
  - Add connections to nearest neighbors for source and target
  - Use graph-traversal algorithm to find a path in the roadmap



PRM – Implementation details

- Understand the C-space
  - Distance metric
- Tools
  - Collision detector, local planner, (k)nearest neighbor
- Sampling strategy
- Path post-processing (smoothing, shortcuts)

PRM - Probabilistic completeness

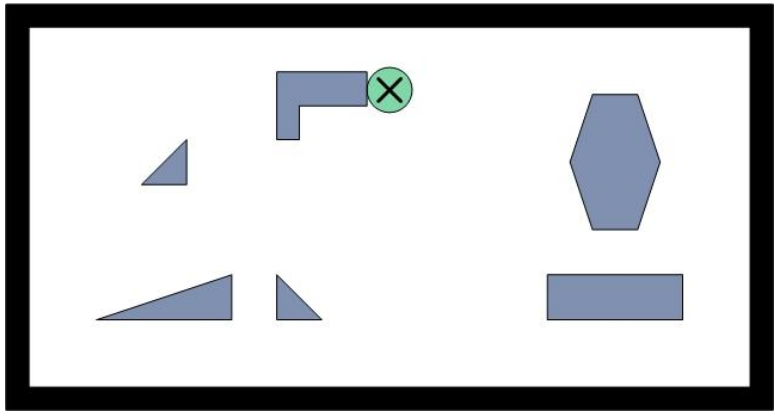
- An algorithm is probabilistically complete if the probability that the algorithm will return a solution tends to one as the time tends to infinity.
- PRM is probabilistically complete
  - Proof – sketch on the board (if time permits)

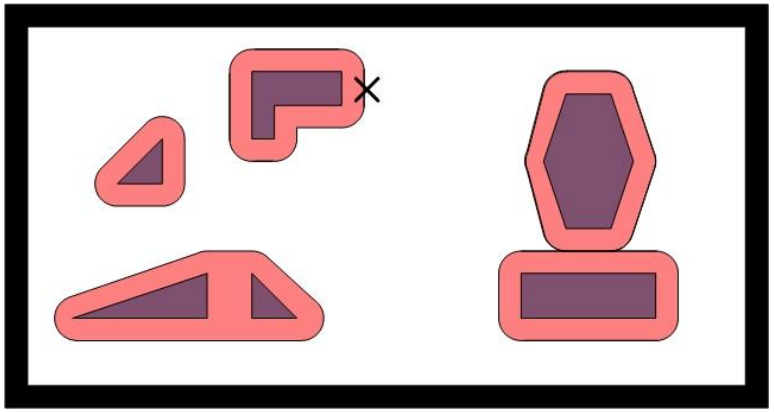
High-quality sampling-based planning

- What is a high-quality path?
  - Short, high-clearance, low-energy, smooth, a combination of the above…
- In general PRM may return a path which is very bad…
- There are heuristics to improve PRM path quality and variations to produce asymptotically optimal paths

Combinatorial Motion Planning

- Analytically compute an explicit combinatorial representation of C_{free}
  - Using critical hyper-surfaces*



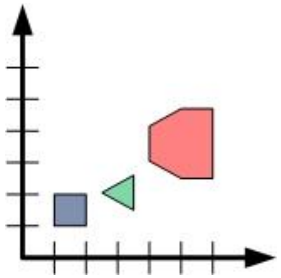


*Schwartz, Sharir 83: On the "piano movers" problem. II. General techniques for computing topological properties of real algebraic manifolds

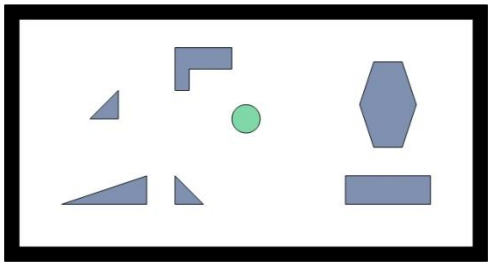
Combinatorial Motion Planning (cont.)

- Minkowski Sums –

$$P \oplus Q = \{p + q |\ p \in P, q \in Q\}$$



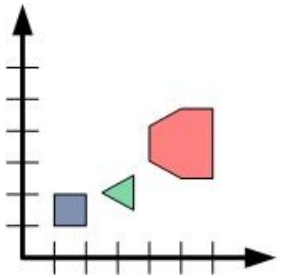
- Allow representation of the configuration space of a translating robot



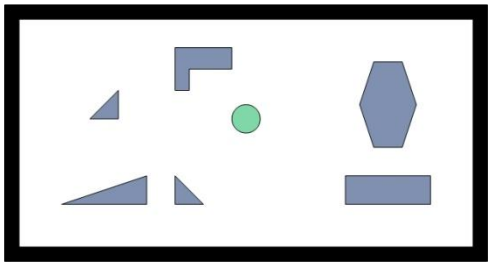
Combinatorial Motion Planning (cont.)

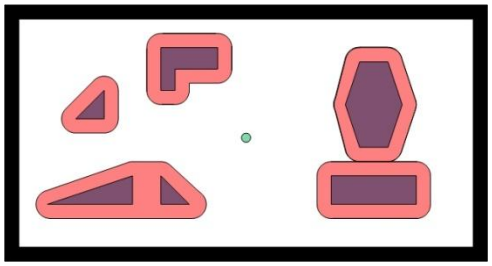
- Minkowski Sums –

$$P \oplus Q = \{p + q |\ p \in P, q \in Q\}$$



- Allow representation of the configuration space of a translating robot

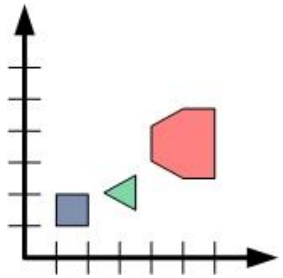




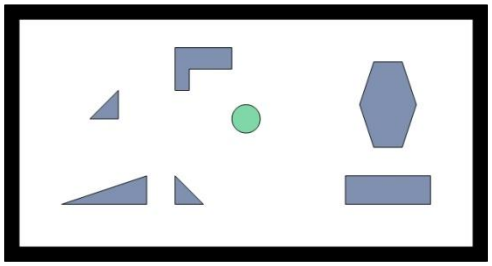
Combinatorial Motion Planning (cont.)

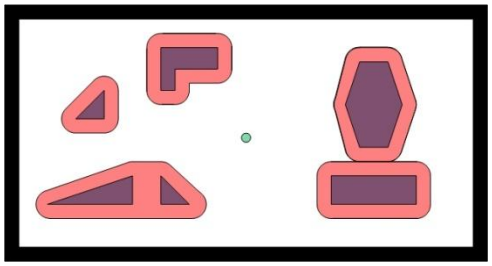
- Minkowski Sums –

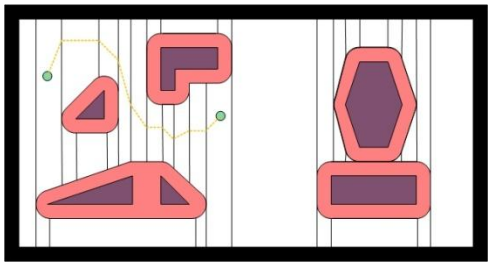
$$P \oplus Q = \{p + q |\ p \in P, q \in Q\}$$



- Allow representation of the configuration space of a translating robot







Advantages and Limitations of Approaches

Probabilistic planning

- ✓ **Easy** to implement
- ✓ Applicable to **high-dimension** C-spaces
- ✗ Sensitive to **tight passages**

Combinatorial planning

- ✗ Complex implementations
- ✗ Exponential in the number of degrees of freedom*
- ✓ **Analytic complete** representation

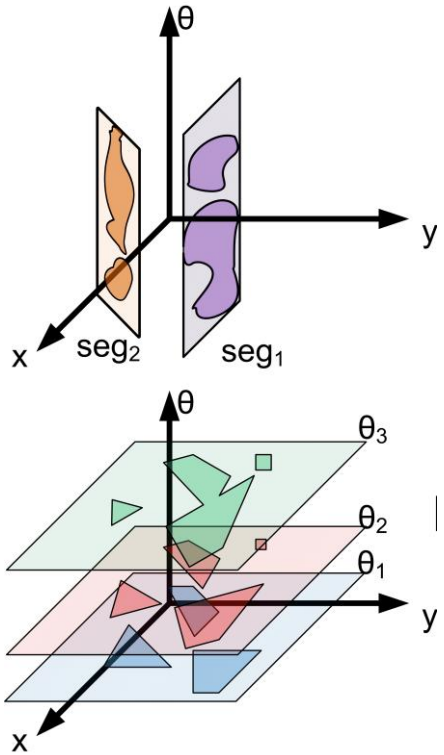
*Reif 79: Complexity of the mover's problem and generalizations

Outline

- Sampling-based motion-planning
- Motion Planning via Manifold Samples (MMS)
- MMS for 3DoF polygonal robot

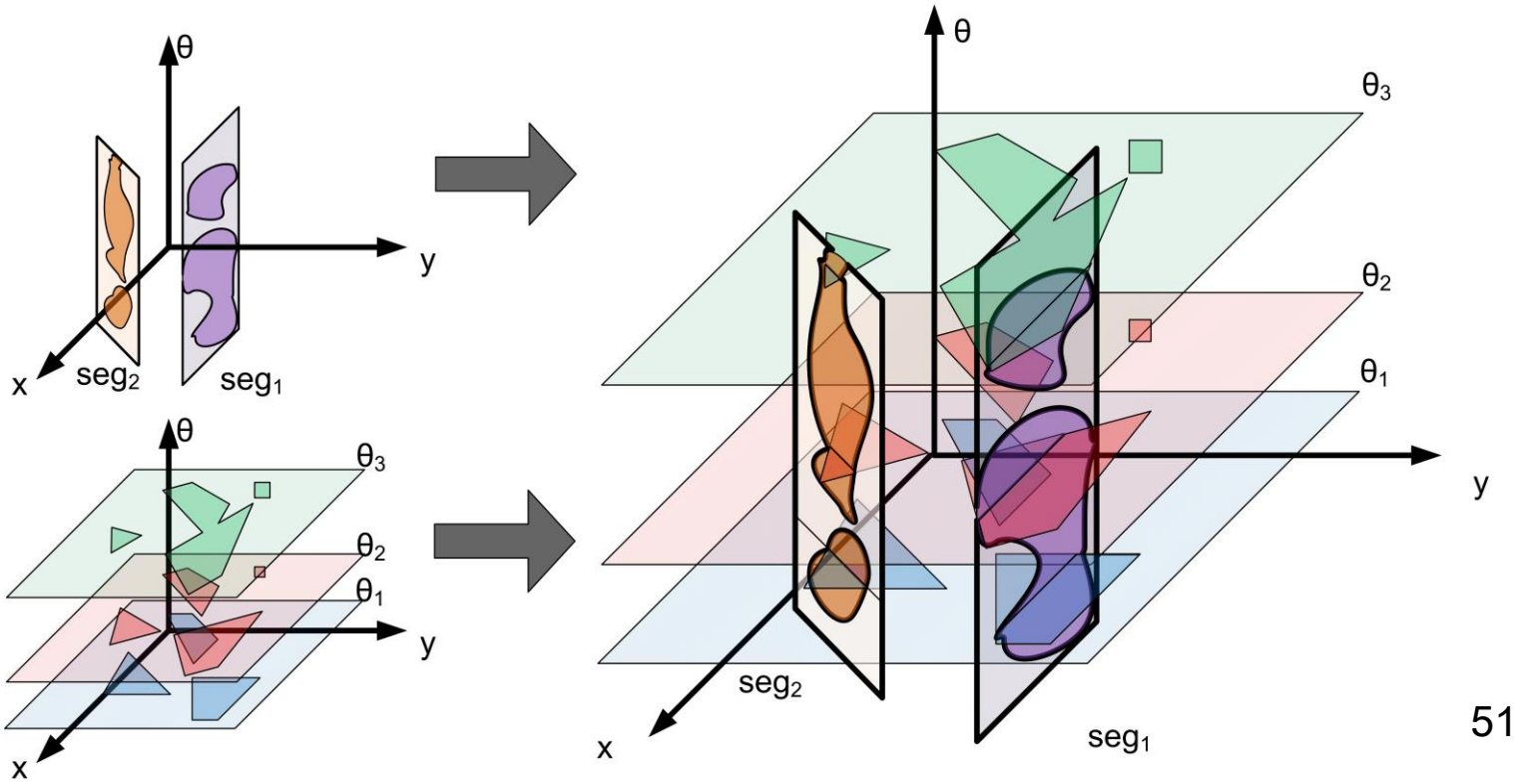
Overview

- Sampling-based multi-query planner
- Samples are entire manifolds of low dimensions
- Manifolds are **decomposed** analytically into cells
  - A cell in C_{free} is a **Free Space Cell** (FSC)



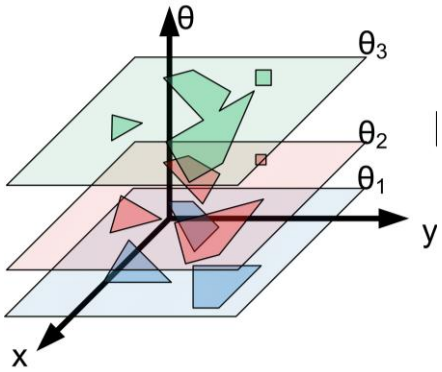
Overview

- **Preprocessing stage -** construct graph G = (V,E)
  - V – FSCs
  - E – Intersecting FSCs
- **Query stage**



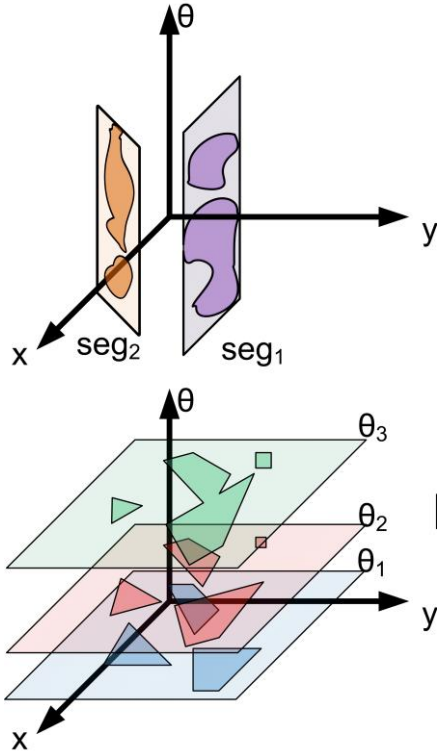
Exploration Vs. Connection

- Manifold samples add
  - **vertices** (new connected components)



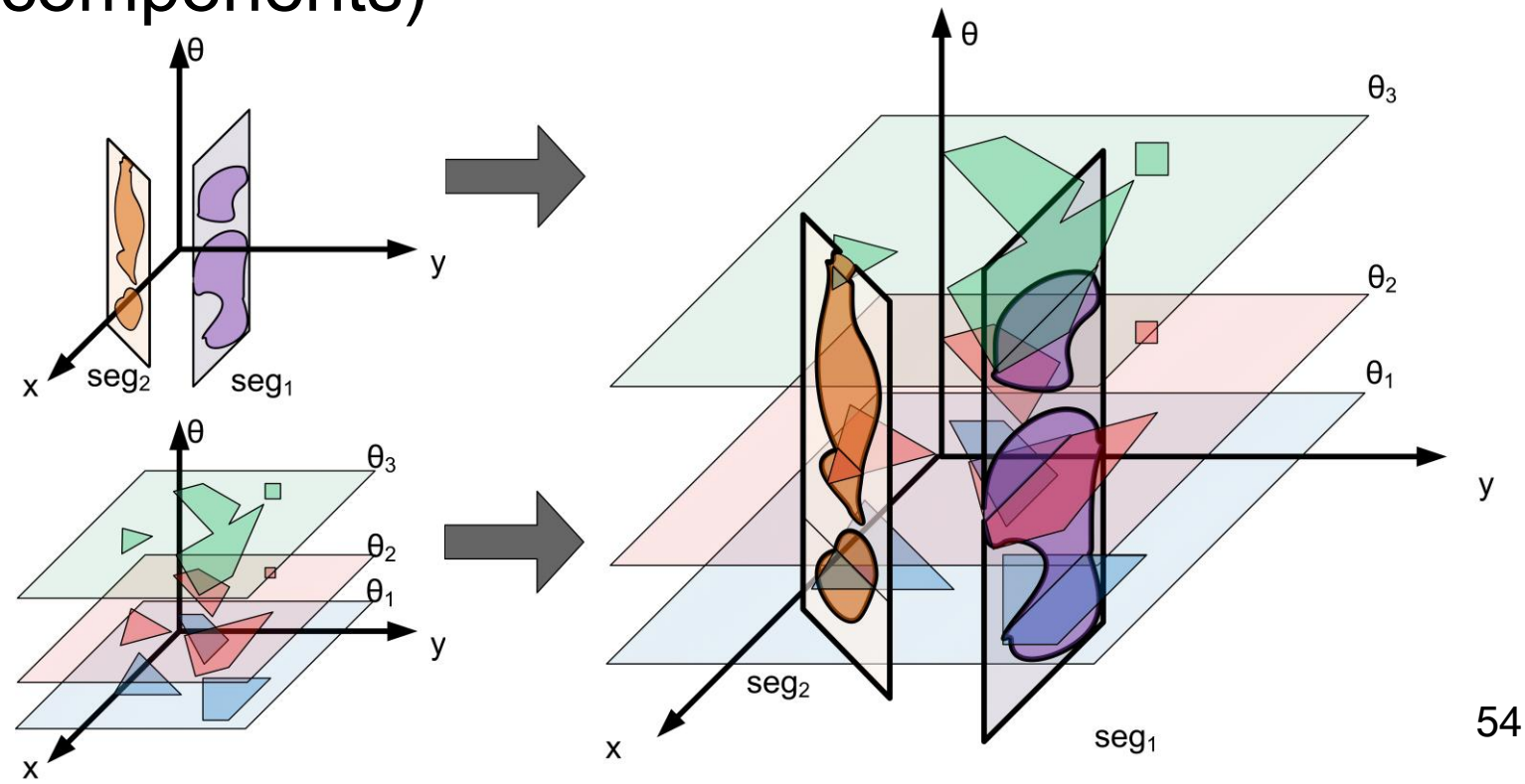
Exploration Vs. Connection

- Manifold samples add
  - **vertices** (new connected components)
  - **edges** (connect existing connected components)



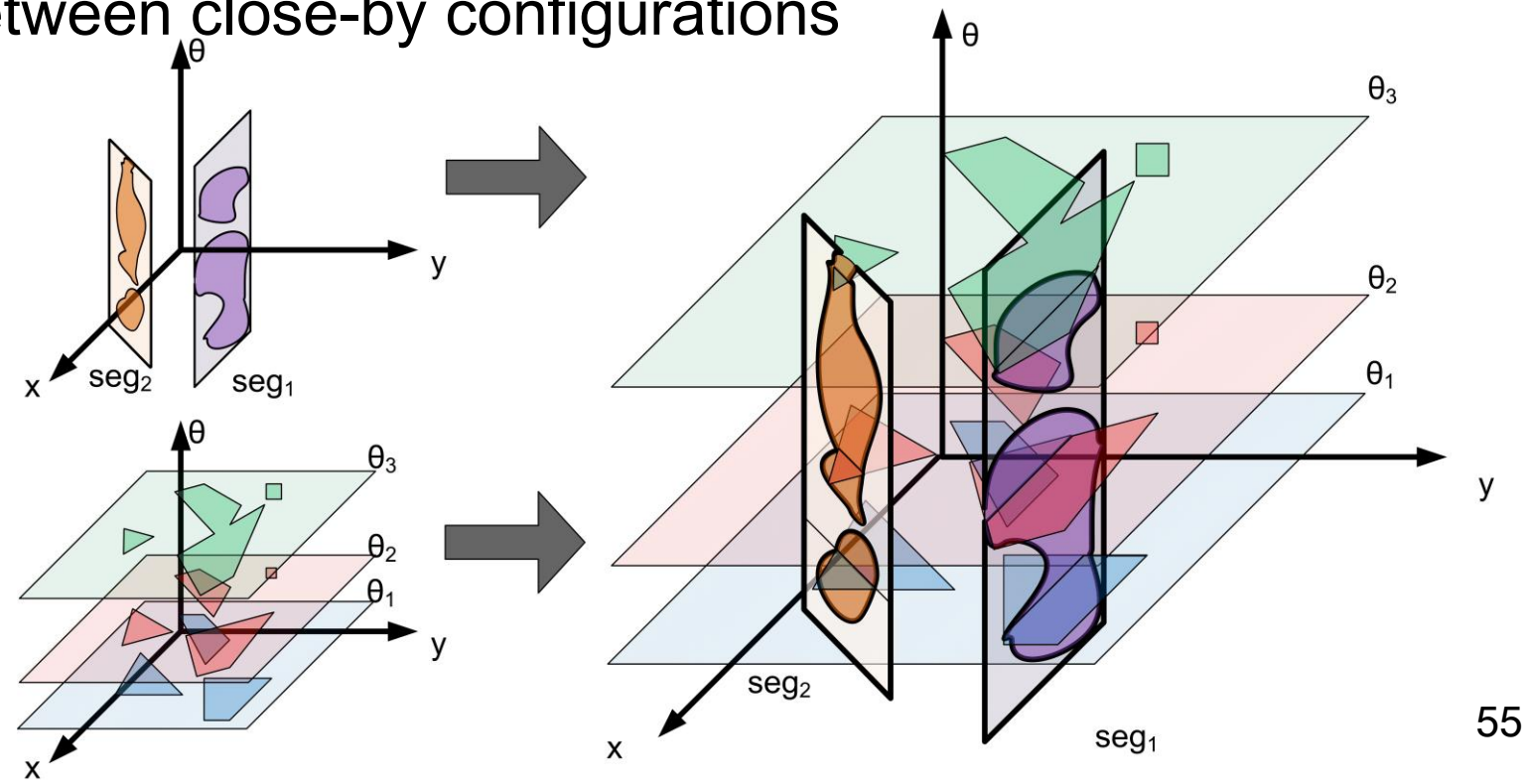
Exploration Vs. Connection

- Manifold samples add
  - **vertices** (new connected components)
  - **edges** (connect existing connected components)



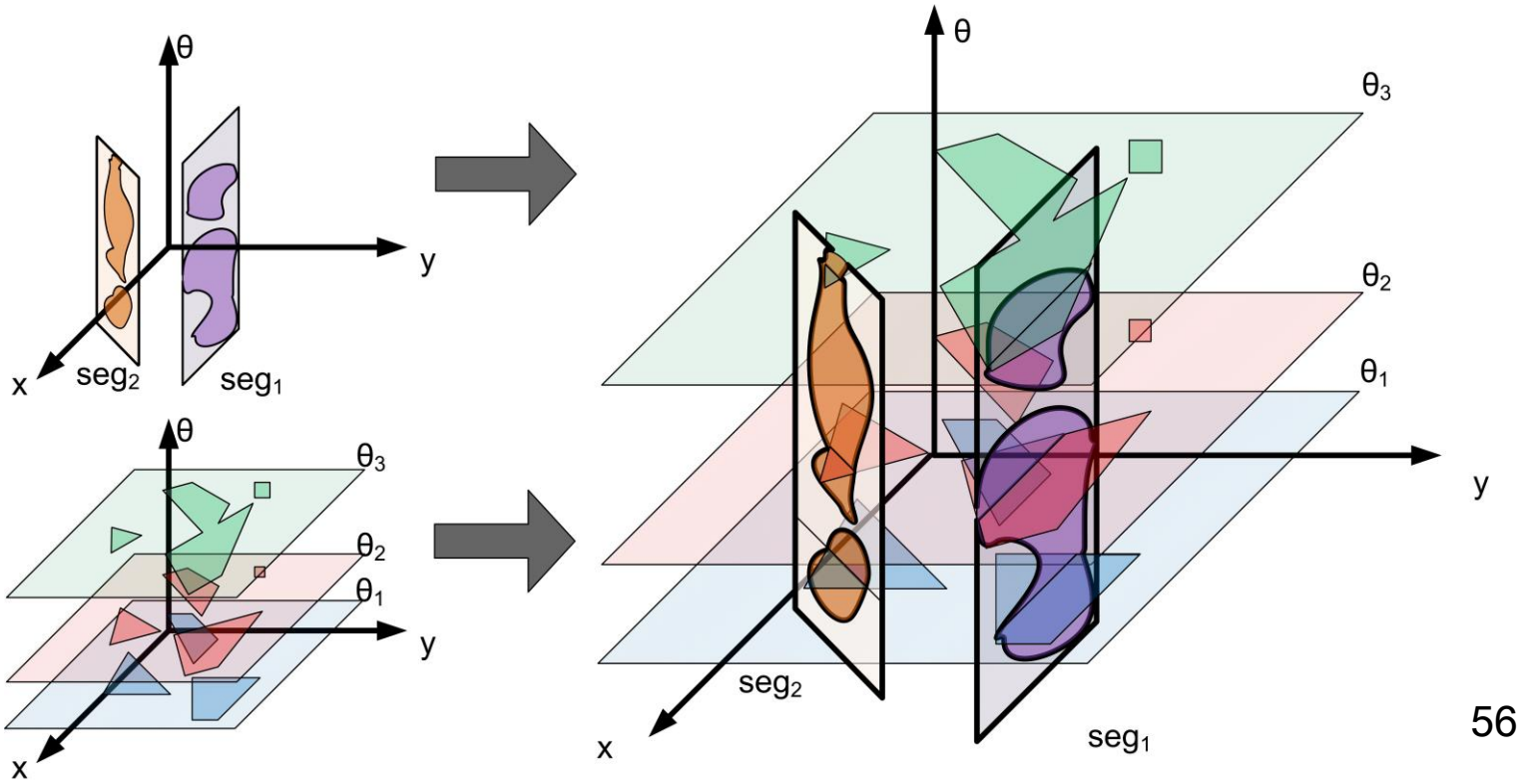
Desired Properties of Manifolds

- **Simplicity**: Easy representation, construction and decomposition
- **Covering**: Manifolds should be dense
- **Spanning**: Manifolds should allow local connections between close-by configurations



Comparison With PRM

	PRM	MMS
Sample type	Point	Manifold
Decomposition	Collision detector	Analytic primitive
Node type	Point	FSC
Node connection	Interpolation	Analytic (intersection)
Data structure	Roadmap graph	Geometric intersection graph of FSCs

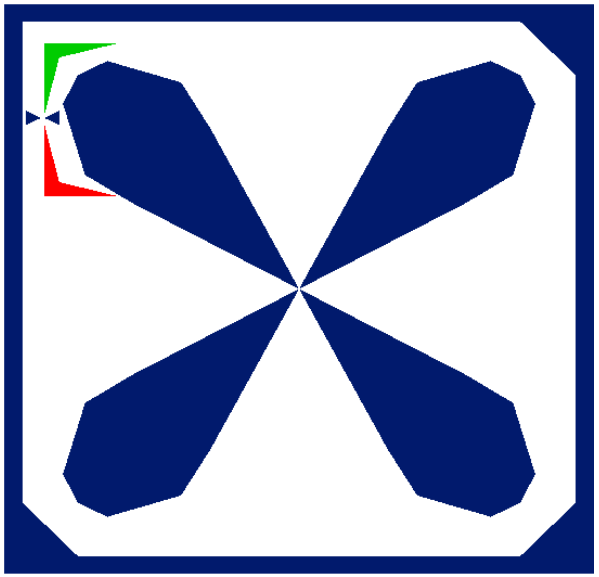


Outline

- Sampling-based motion-planning
- Motion Planning via Manifold Samples (MMS)
- MMS for 3DoF polygonal robot

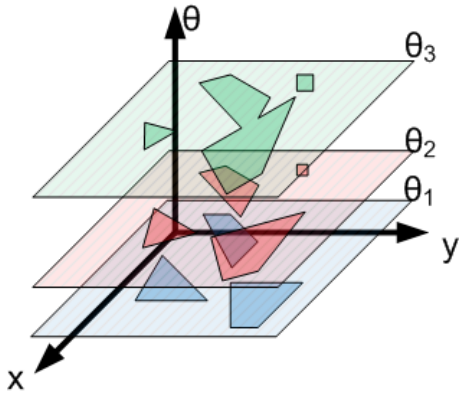
The Setting

- Two-dimensional polygonal robot R
- Three-dimensional configuration space
  - translation and rotation



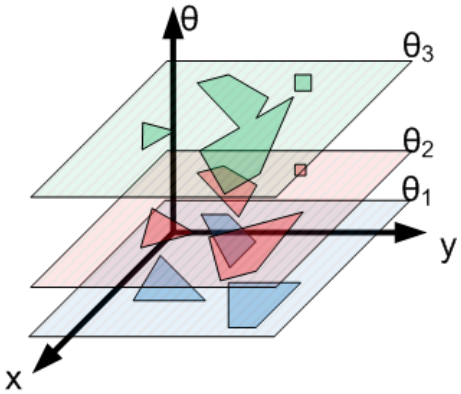
Families of Manifolds

- **Fixed rotation angle**
  - Horizontal planes
  - Computation via Minkowski sums,

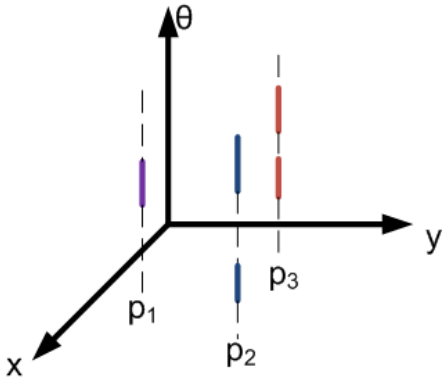


Families of Manifolds

- **Fixed rotation angle**
  - Horizontal planes
  - Computation via Minkowski sums,



- **Fixed reference point**
  - Vertical lines
  - Computed analytically via critical angles



Fixed Rotation Angle (details)

- (Thm) Let R(x,y) be a robot placed at x,y and P be an obstacle, then R intersects P iff

$$(x,y) \in P \oplus -R(0,0)$$

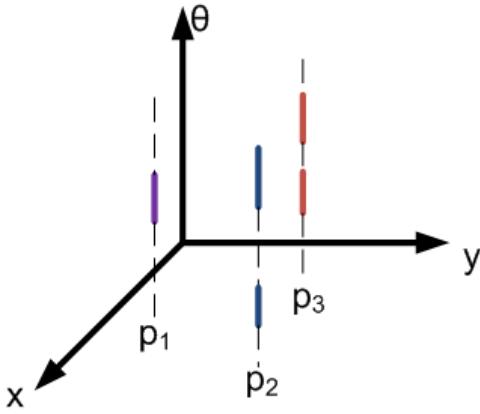
- If R(x,y) intersects P
  - Let q be the intersection point
  - $q \in R(x,y) \Rightarrow q - (x,y) \in R(0,0) \Rightarrow -q+(x,y) \in -R(0,0)$
  - $q \in P$
  - Thus, $(x,y) \in P \oplus -R(0,0)$
- If $(x,y) \in P \oplus -R(0,0)$
  - There are points $-r \in R(0,0)$, $p \in P$ s.t.
  - $(x,y) = p-r \Rightarrow p = (x,y) + r$
  - =>The robot placed at (x,y) intersects P

Fixed Rotation Angle (details)

- For a robot R with its reference point at the origin and an obstacle O, the forbidden space is represented by $P \oplus -R(0,0)$

Fixed Reference Point (details)

- Parameterization:
  - $\alpha \in [0, 1]$ reference point on segment



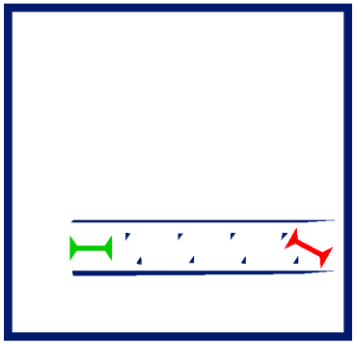
- Parameterized **critical angles** are in the form of algebraic numbers*

*Algebraic number - a number that is a root of a non-zero polynomial in one variable with rational coefficients

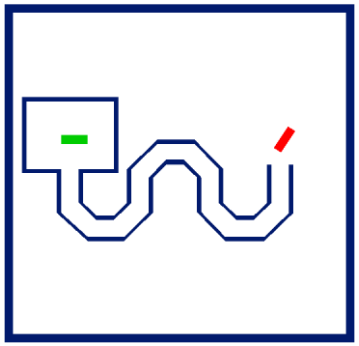
Experimental Results

- Scenarios

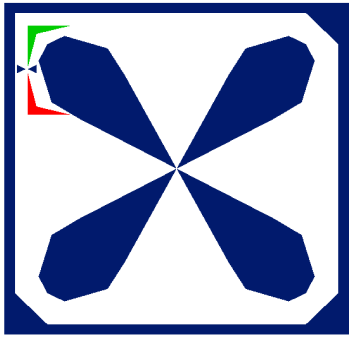
Tunnel



Snake



Flower

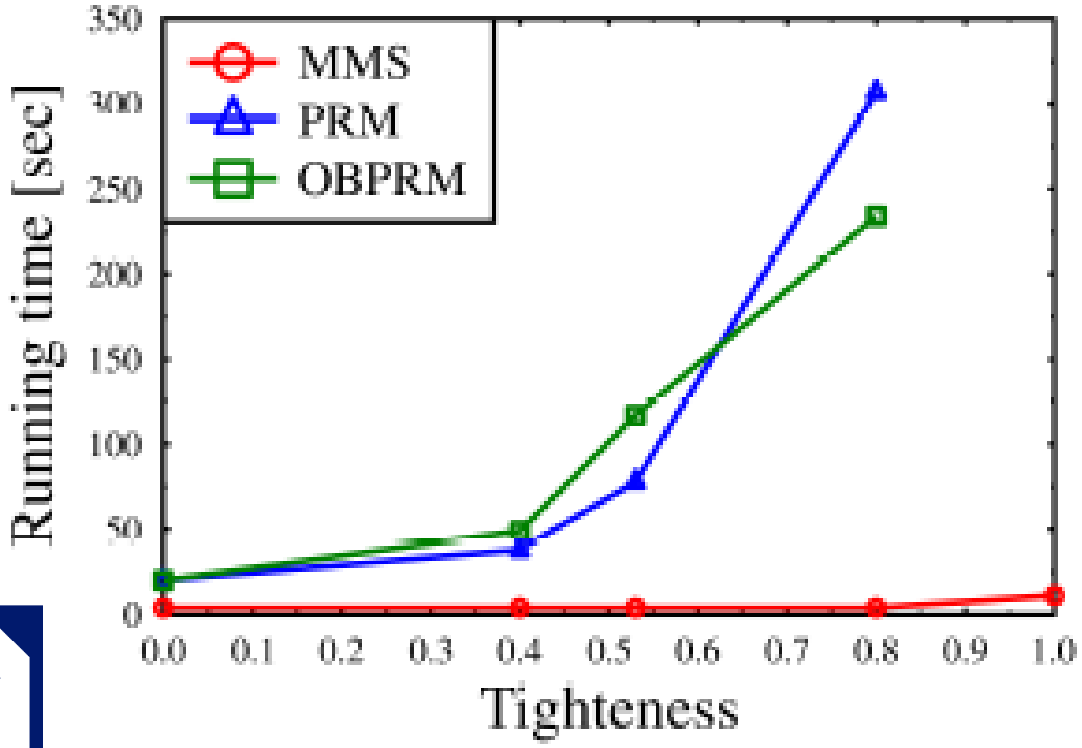


- OOPSMP PRM Implementation

Scenario	MMS			PRM			OBPRM		
	n_θ	n_s	t[sec]	k	res	t[sec]	k	res	t[sec]
Tunnel	20	128	**21**	10	0.005	**114**	10	0.01	**134**
Snake	40	256	**35**	10	0.02	**264**	10	0.01	**247**
Flower	40	256	**4**	20	0.02	**20**	14	0.01	**20**

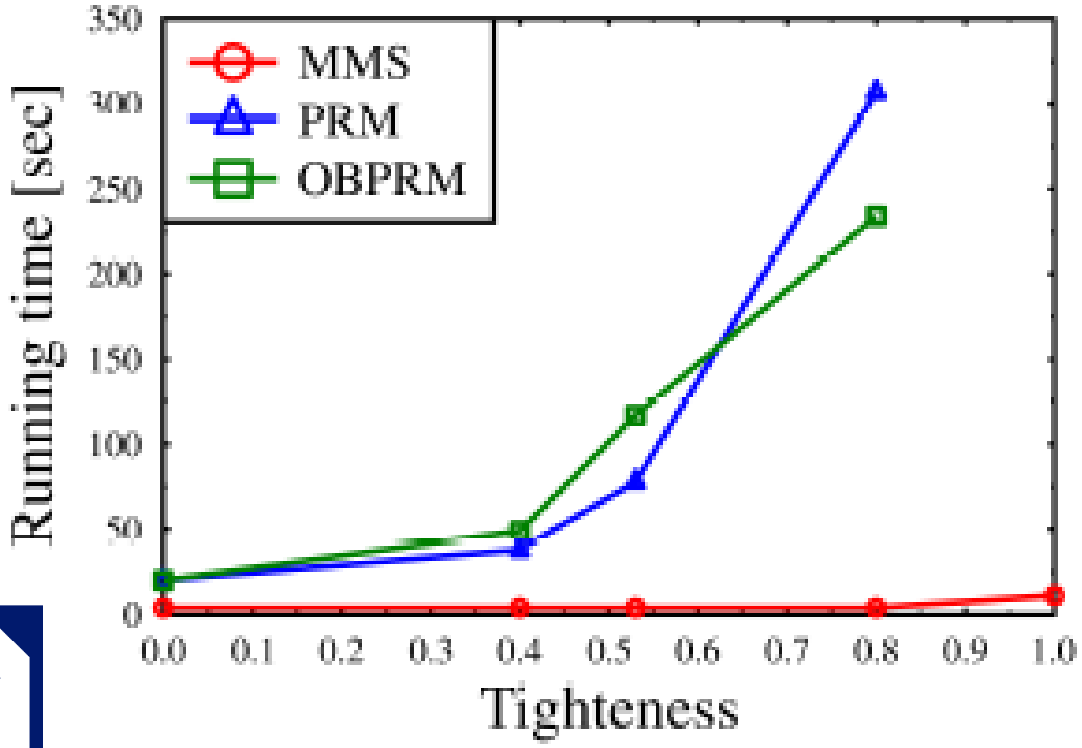
Experimental Results

- Tightening the configuration space



Experimental Results

- Tightening the configuration space



20-fold speedup