Coordinating the Motion of Discs

Software Workshop:
High-Quality Motion Paths for Robots (and Other Creatures)

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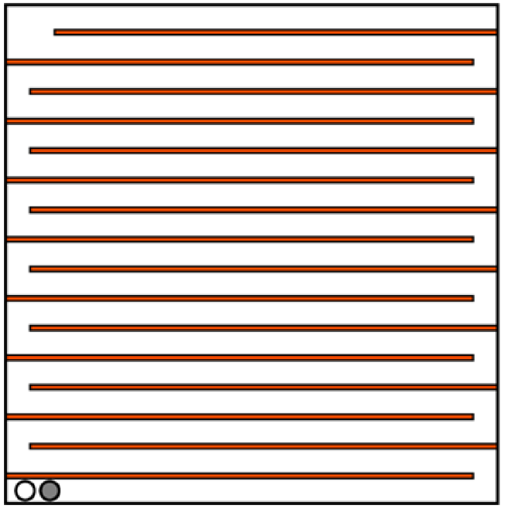
Project

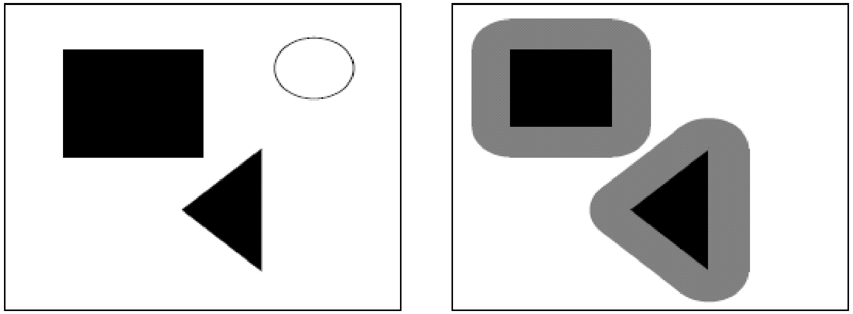
Given k discs in the plane moving among polygonal obstacles, with free start and goal placement for each disc, decide if a motion path exists for all the discs from start to goal while avoiding the obstacles and each other



Distinctive features:

- small k
- high accuracy
- web interface

The case of one disc
of radius r

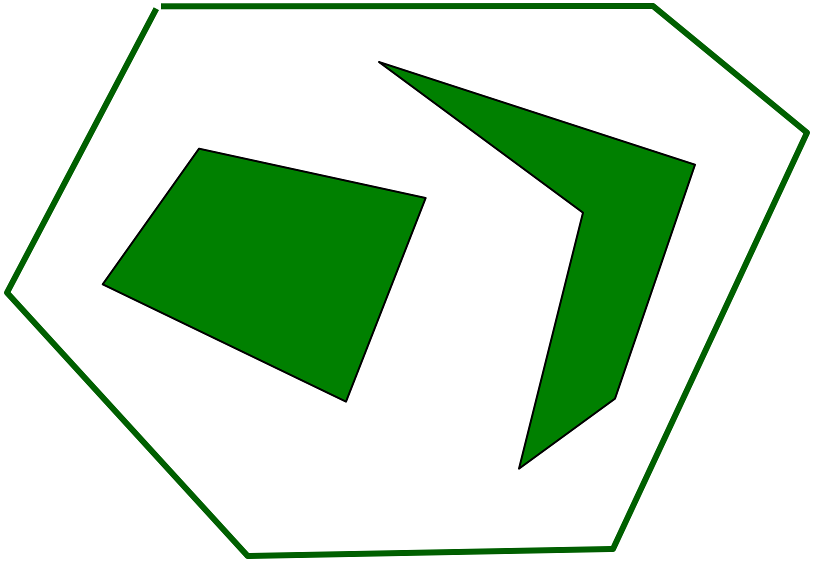


- convention: a disc center is its reference point
- the configuration space is two dimensional
- expanded obstacles are so-called offset polygons (or r-offset polygons)
- the free space is the complement of the union of the offset polygons for all obstacle polygons

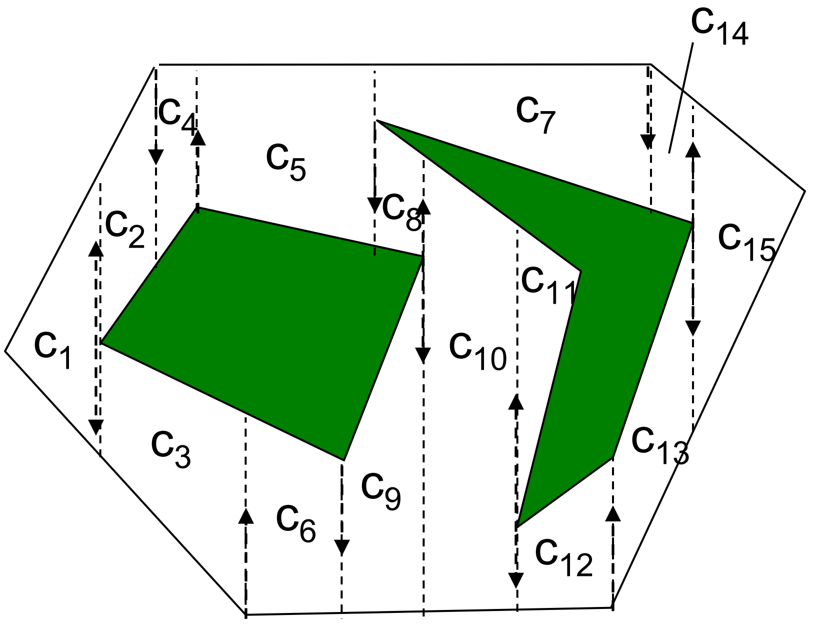
The case of one disc, cont'd

- major issues:
  - how to represent the free space
  - how to compute it
  - how to extract a solution from the representation

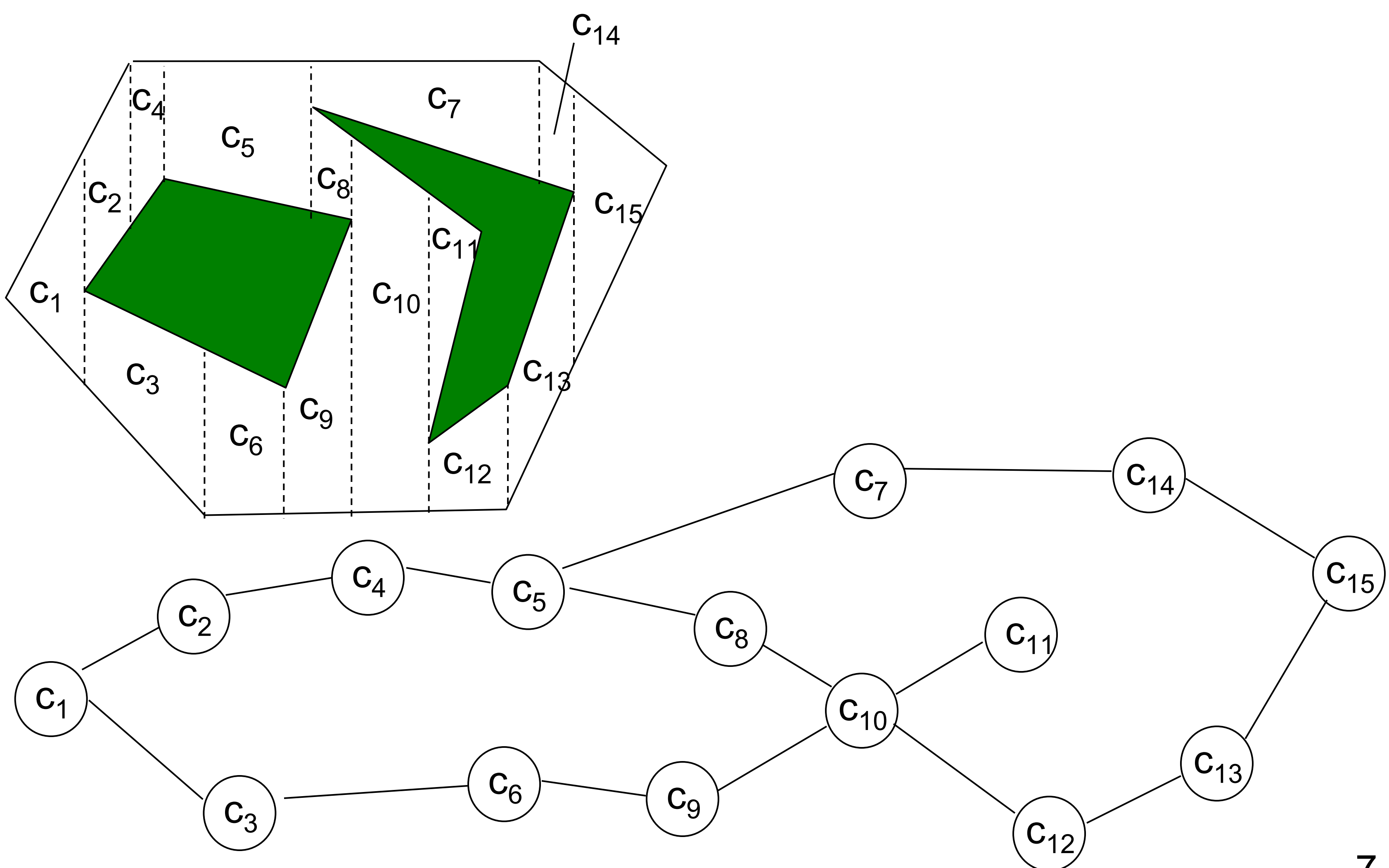
Point robot among polygonal obstacles



Trapezoidal decomposition

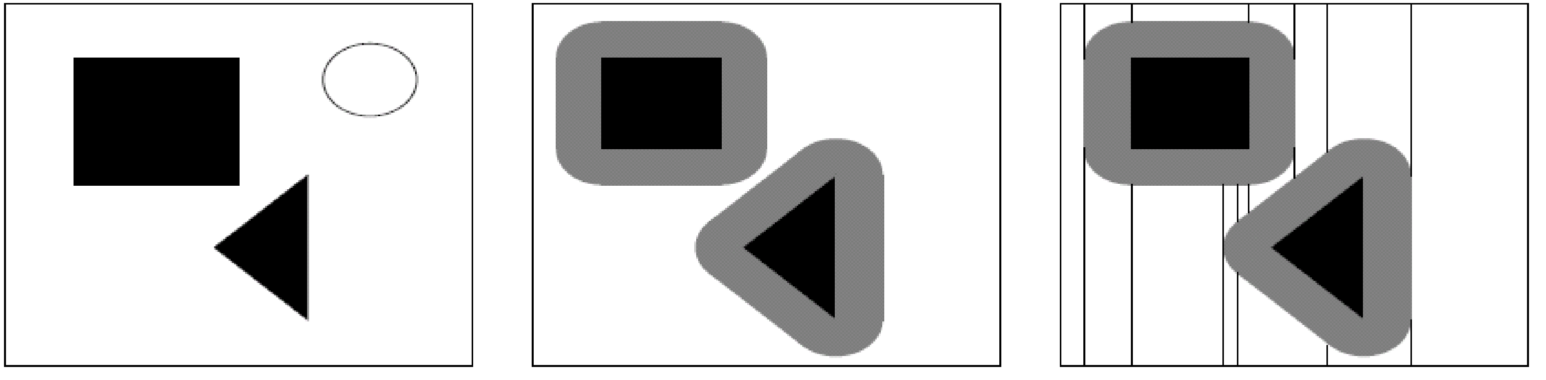


Connectivity graph



The case of one disc, cont'd

- the free space is similar and can be similarly handled
- the only difference is that now faces are bounded not only by segments but also by circular arcs
- trapezoidal decomposition is adapted by adding vertices through vertical tangencies of circular arcs

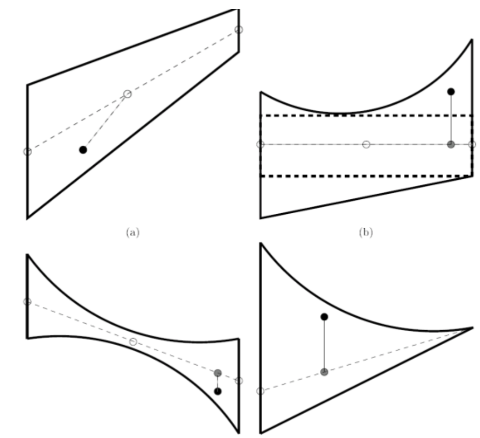


How to compute it

- CGAL does almost everything for you:
  - offset polygons, either exact or finely approximated
  - union, complement, trapezoidal decomposition (aka vertical decomposition)

How to extract a path

- if you just wish to extract a path, then there is a standard way by putting milestones in the center of cells and in the centeras of vertical walls (some caution is needed since the cells are not necessarily convex)



- extracting good paths requires more work and creativity (some more details will be given in Lesson IV)

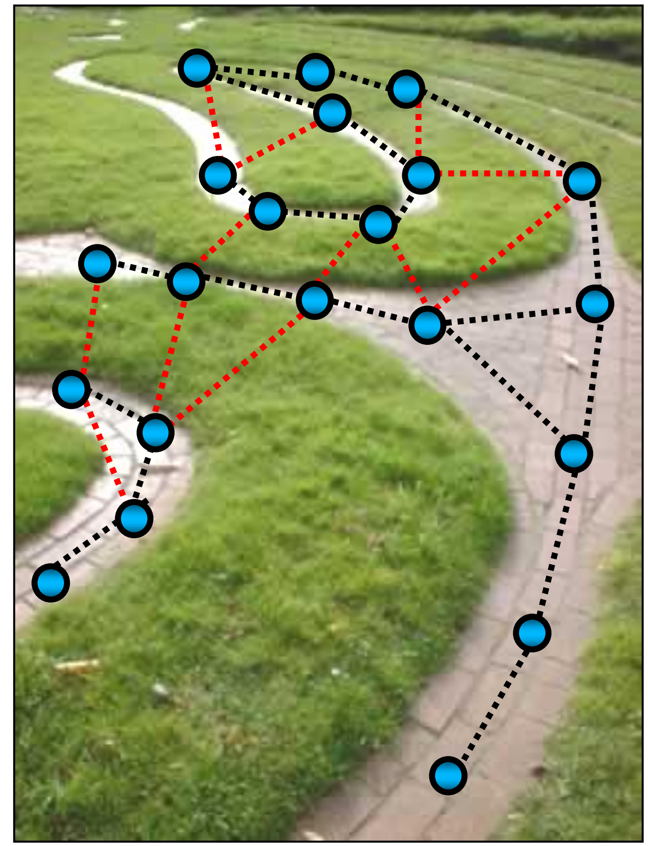
The case of two discs

- the configuration space is 4-dimensional
- options
  - exact solution – not infeasible but will take tremendous effort with current technology
  - sampling-based solution

sampling-based solution, reminder

basic PRM (Probabilistic Road-Map) algorithm in a nutshell

- randomly sample *n* valid robot configurations (“milestones”)
- connect close-by configurations by dense sampling (“local-planning”)
  - discard invalid edges

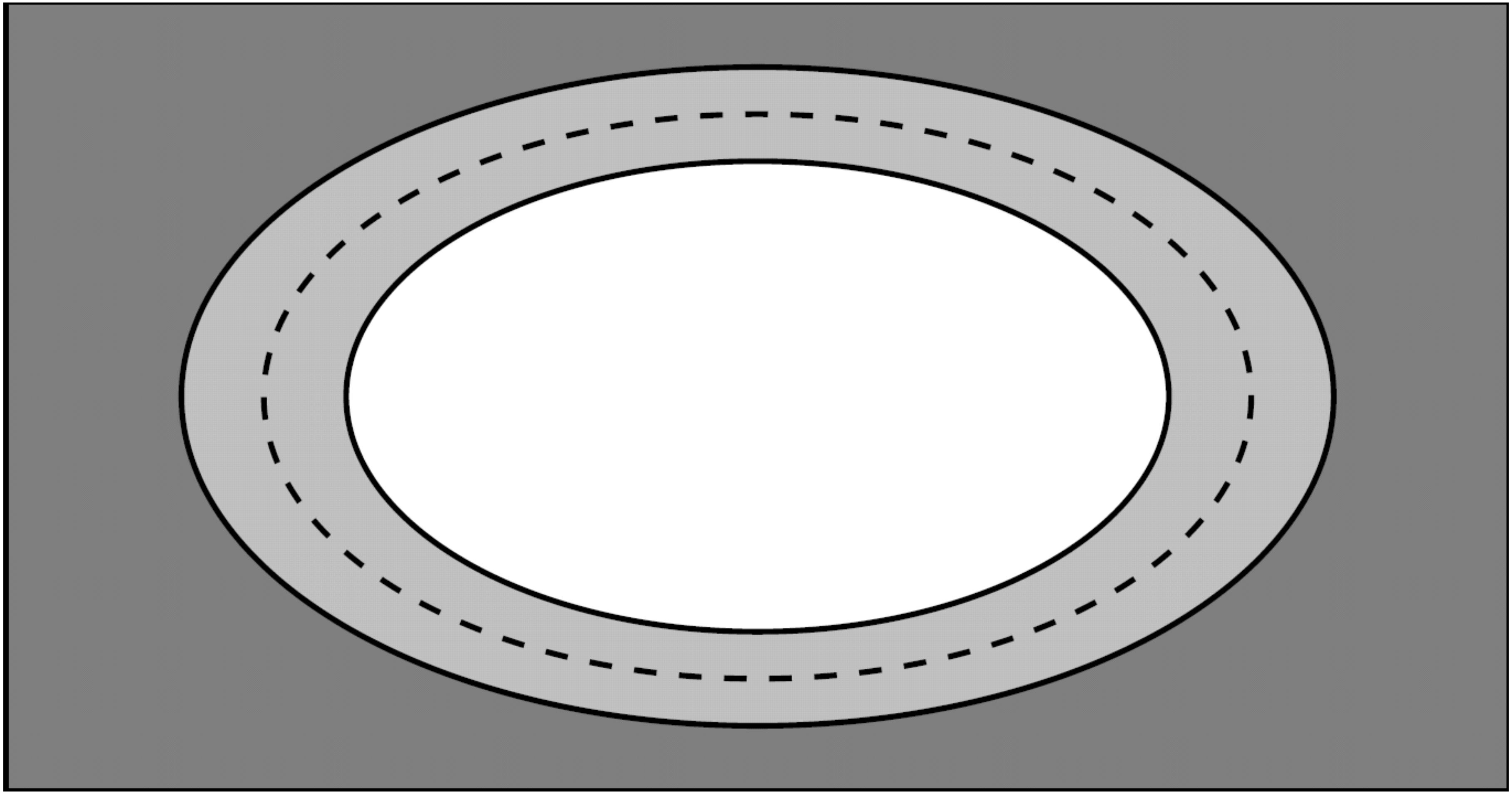


Note:
For simplicity, configuration space and workspace are identical, in this example

The case of two discs

- the configuration space is 4-dimensional
- options
  - exact solution – not infeasible but will take tremendous effort with current technology
  - sampling-based solution: easy to implement but ineffective in tightly cluttered environments
  - hybrid solution

The spirit of the hybrid solution



Sketch of a hybrid solution

- compute the free space for each disc separately, and use VertDeco to represent it, V1 and V2
- take the Cartesian product of V1 x V2, and extend it into a connectivity graph using PRM
- many technicalities and room for optimization

A hybrid solution, more information

Shai Hirsch and Dan Halperin
Hybrid Motion Planning: Coordinating Two Discs Moving Among Polygonal Obstacles in the Plane
Proc. 5th Workshop on Algorithmic Foundations of Robotics (WAFR), Nice, 2002, pp, 225-241.

http://acg.cs.tau.ac.il/projects/internal-projects/hybrid-motion-planning-coordinating-2-discs/project-page

Hybrid solution, notes

- we will supply various ready-made software components, including the solution for one disc, a PRM implementation for 2-discs in CGAL, a script to produce mazes
- concrete challenge: beat the CGAL PRM-implementation on the mazes
- web interface
- combination of building a software system/application with algorithmic creativity

THE END