Motion Planning via Manifold Samples* (MMS)

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*Joint work with Michael Hemmer, Barak Raveh and Dan Halperin

TEL AVIV UNIVERSITY

Outline

- Background
- Hybrid Motion Planners
- Motion Planning via Manifold Samples (MMS)
- Specific Implementation

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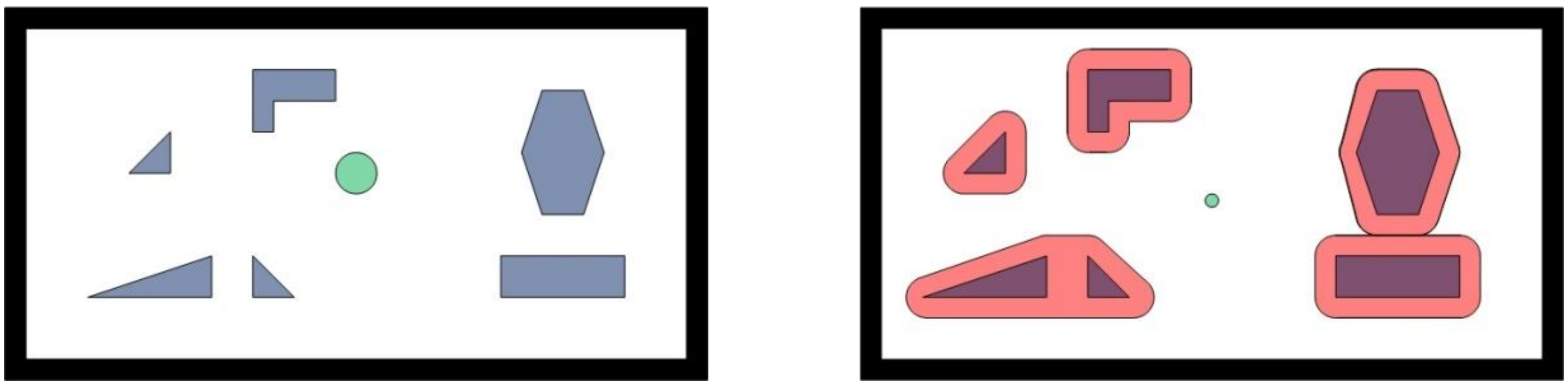
Motion Planning - Definitions

- **Workspace** – A description of the (2D or 3D) world consisting of a **robot** and **obstacles**
- **Configuration Space- (C)** The space of parameters that define the robot's position and orientation in the workspace



Motion Planning - Definitions

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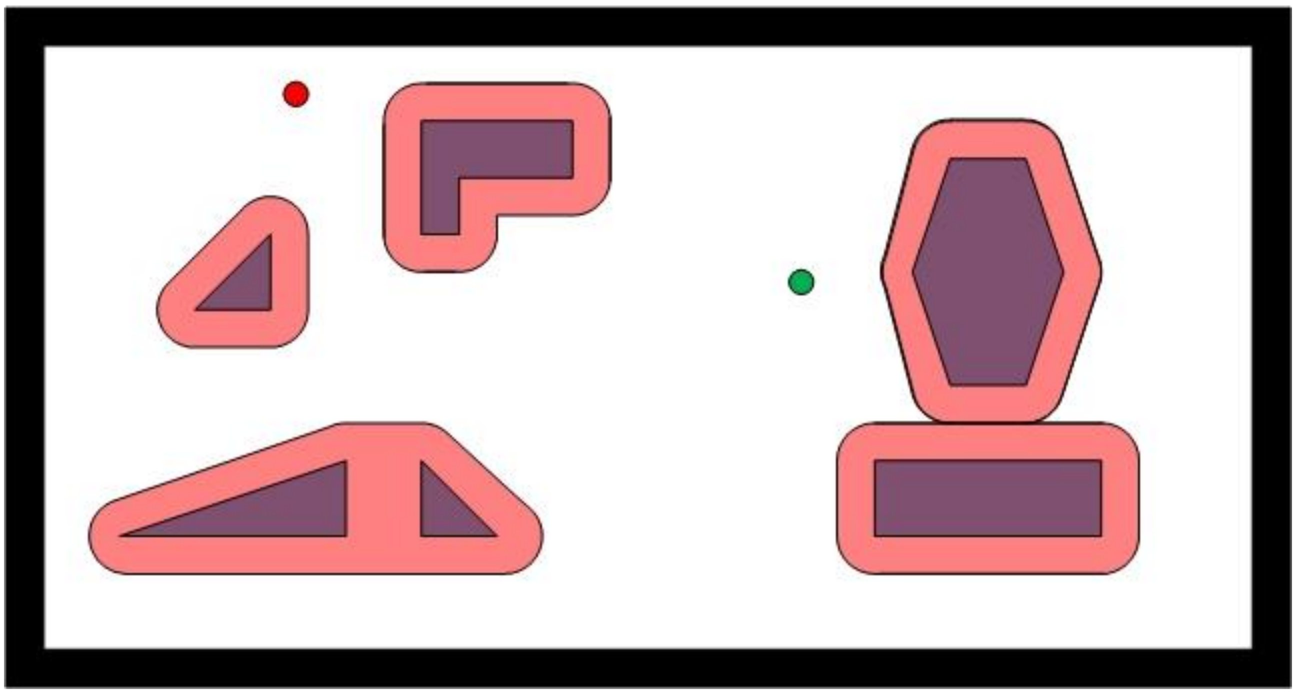
- **Configuration Space- (C)** The space of parameters that define the robot's position and orientation in the workspace
- **Degrees of Freedom-** The minimal number of parameters required to uniquely define a position of the robot
- **Free Space** (C_{free})- Set of collision-free configurations
- **Forbidden Space (C_{forb})-** $C \setminus C_{free}$

Motion Planning - Objective

- Find a path in C_{free} from a free source configuration to a free target configuration



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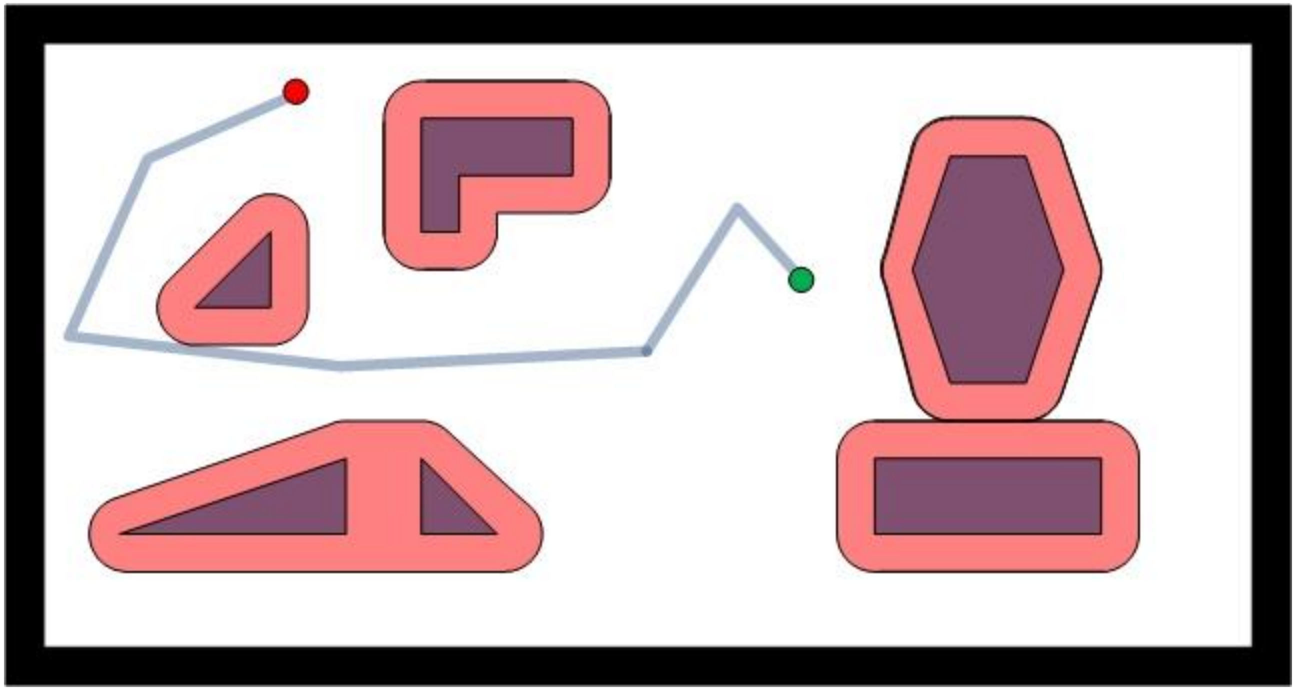


Algorithmic Approaches for Motion Planning

- Sampling-Based Planning
  - Capture connectivity of C_{free} by randomly **sampling** configurations
- Combinatorial Motion Planning
  - **Analytically** compute an explicit combinatorial representation of C_{free}

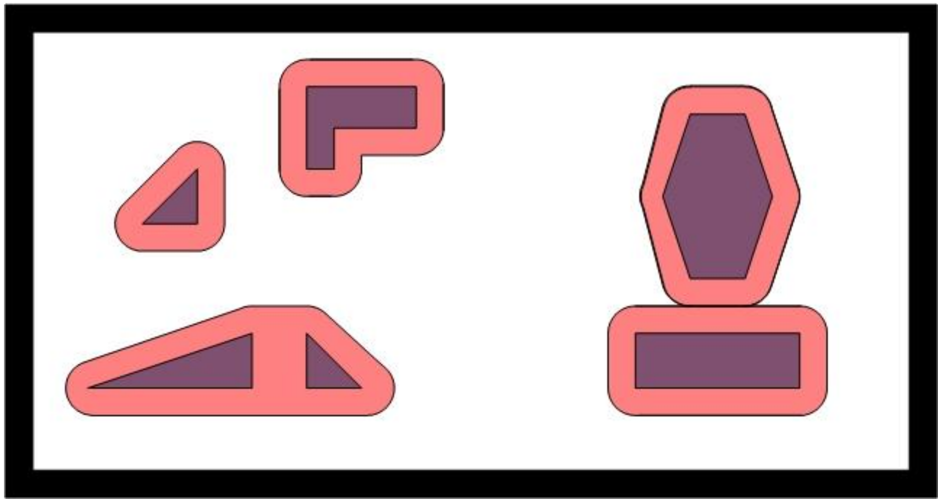
Algorithmic Approaches for Motion Planning

- Sampling-Based Planning
  - Capture connectivity of C_{free} by randomly **sampling** configurations

- Kavraki, Svestka, Latombe, Overmars 96: Probabilistic roadmaps for path planning in high dimensional configuration spaces (PRM)
- LaValle 98: Rapidly-exploring random trees: A new tool for path planning (RRT)
- Hsu, Latombe, Motwani 99: Path planning in expansive configuration spaces (EST)

Sampling-Based Planning - PRM

- Multi query planner
- Preprocesses configuration space into a graph (roadmap)



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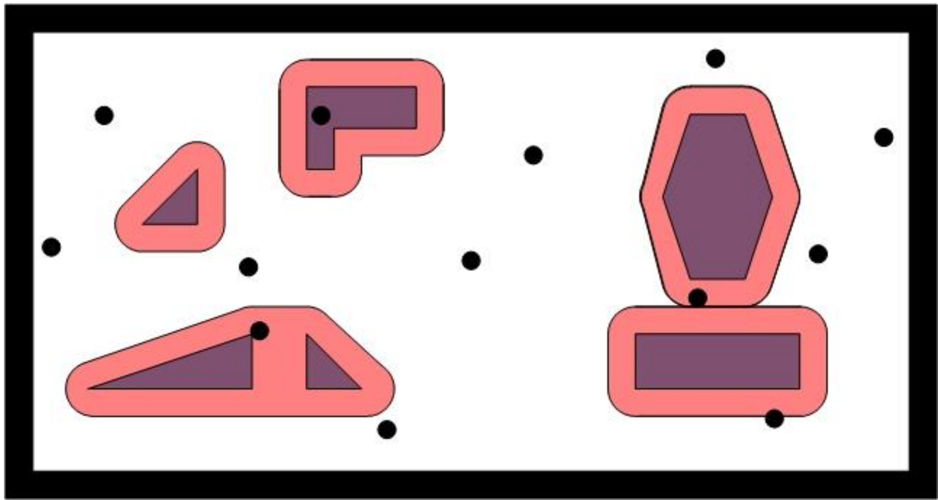

Sampling-Based Planning - PRM

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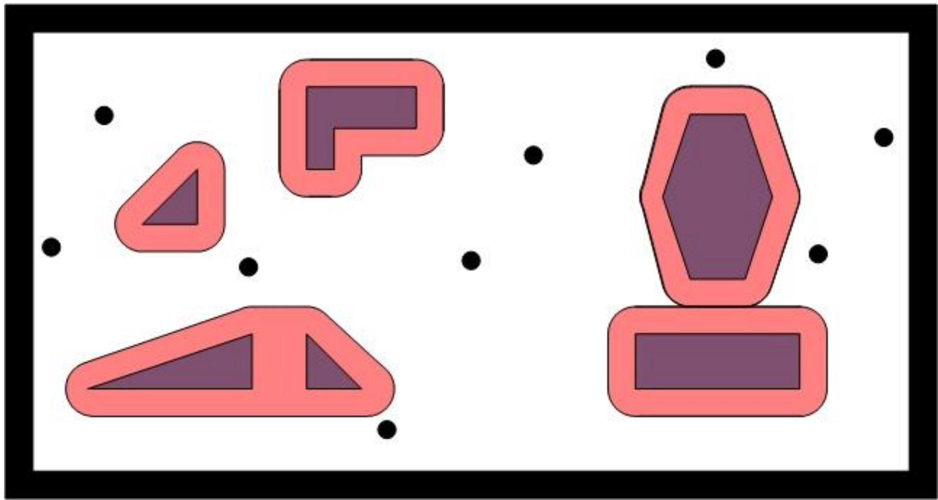
  - Discard invalid configurations



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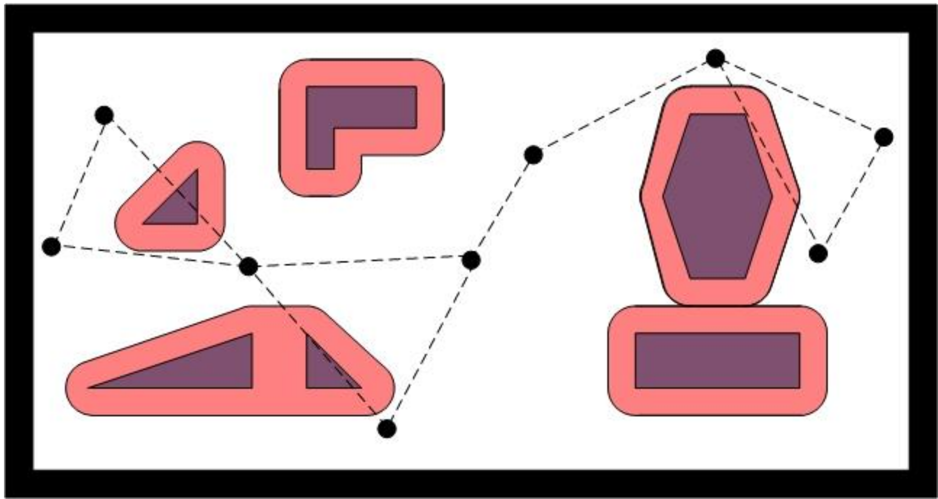
  - Connect close-by configurations by dense sampling ("local-planning")



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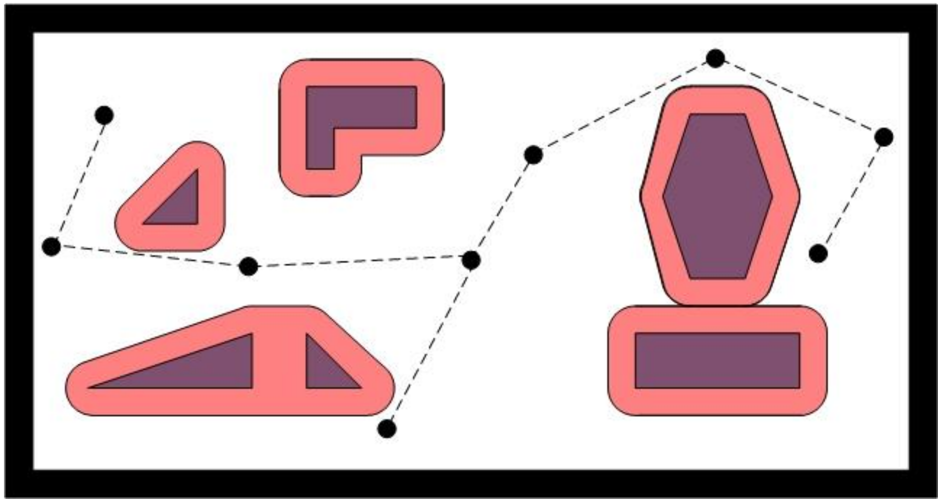
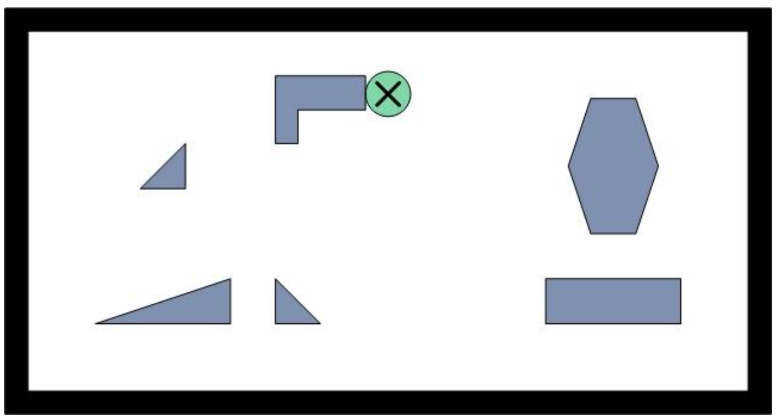
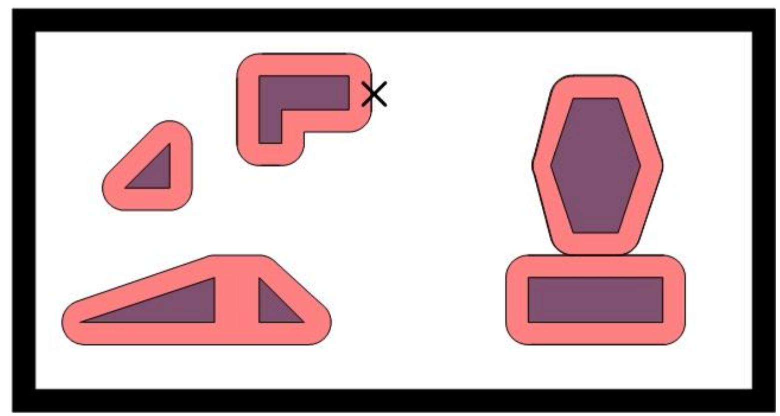
  - Connect close-by configurations by dense sampling ("local-planning")
  - Discard invalid edges



Combinatorial Motion Planning

- Analytically compute an explicit combinatorial representation of C_{free}
  - Using critical hyper-surfaces*



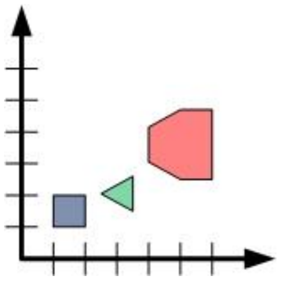


*Schwartz, Sharir 83: On the "piano movers" problem. II. General techniques for computing topological properties of real algebraic manifolds

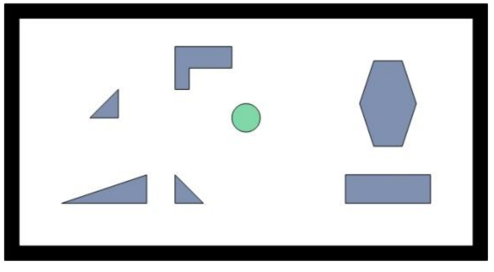
Combinatorial Motion Planning (cont.)

- Minkowski Sums –

  $$P \oplus Q = \{p + q \mid p \in P, q \in Q\}$$



- Allow representation of the configuration space of a translating robot



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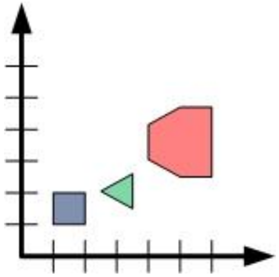

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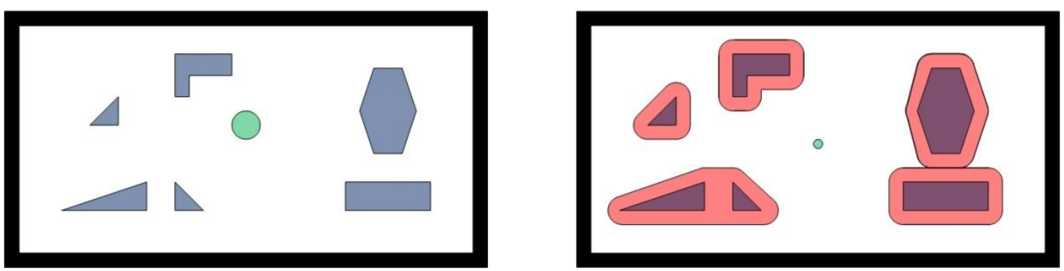

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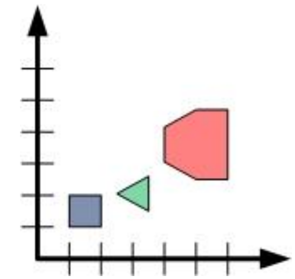

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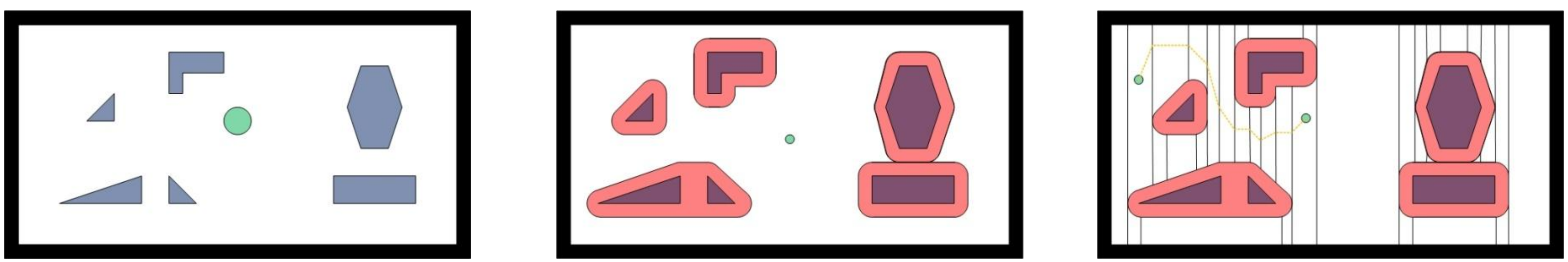

Advantages and Limitations of Approaches

Probabilistic planning

- ✓ **Easy** to implement
- ✓ Applicable to **high-dimension** C-spaces
- ✗ Sensitive to **tight passages**

Combinatorial planning

- ✗ Complex implementations
- ✗ Exponential in the number of degrees of freedom*
- ✓ **Analytic complete** representation

*Reif 79: Complexity of the mover's problem and generalizations

Hybrid Planners

- S. Hirsch and D. Halperin. Hybrid motion planning: Coordinating two discs moving among polygonal obstacles in the plane. WAFR 2002 [HH02]
- Liangjun Zhang, Young J. Kim, and Dinesh Manocha. A hybrid approach for complete motion planning. IROS 2007 [ZKM07]
- Jade Yang and Elisha Sacks. RRT path planner with 3 DOF local planner. ICRA, 2006 [YS06]
- Ming Lien, J.: Hybrid motion planning using Minkowski sums. RSS 2008 [Lie08]

Existing Hybrid Planners - Limitations

- Applicable for low dimensions
- Applicable to specific instances

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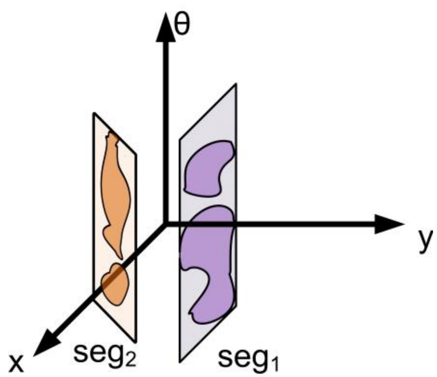
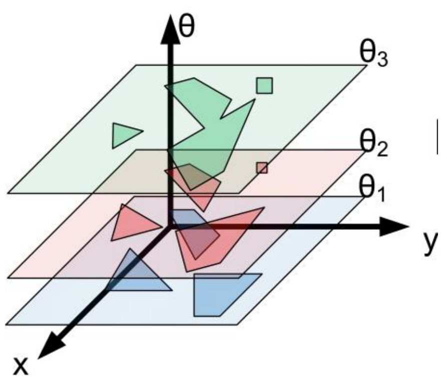
Overview

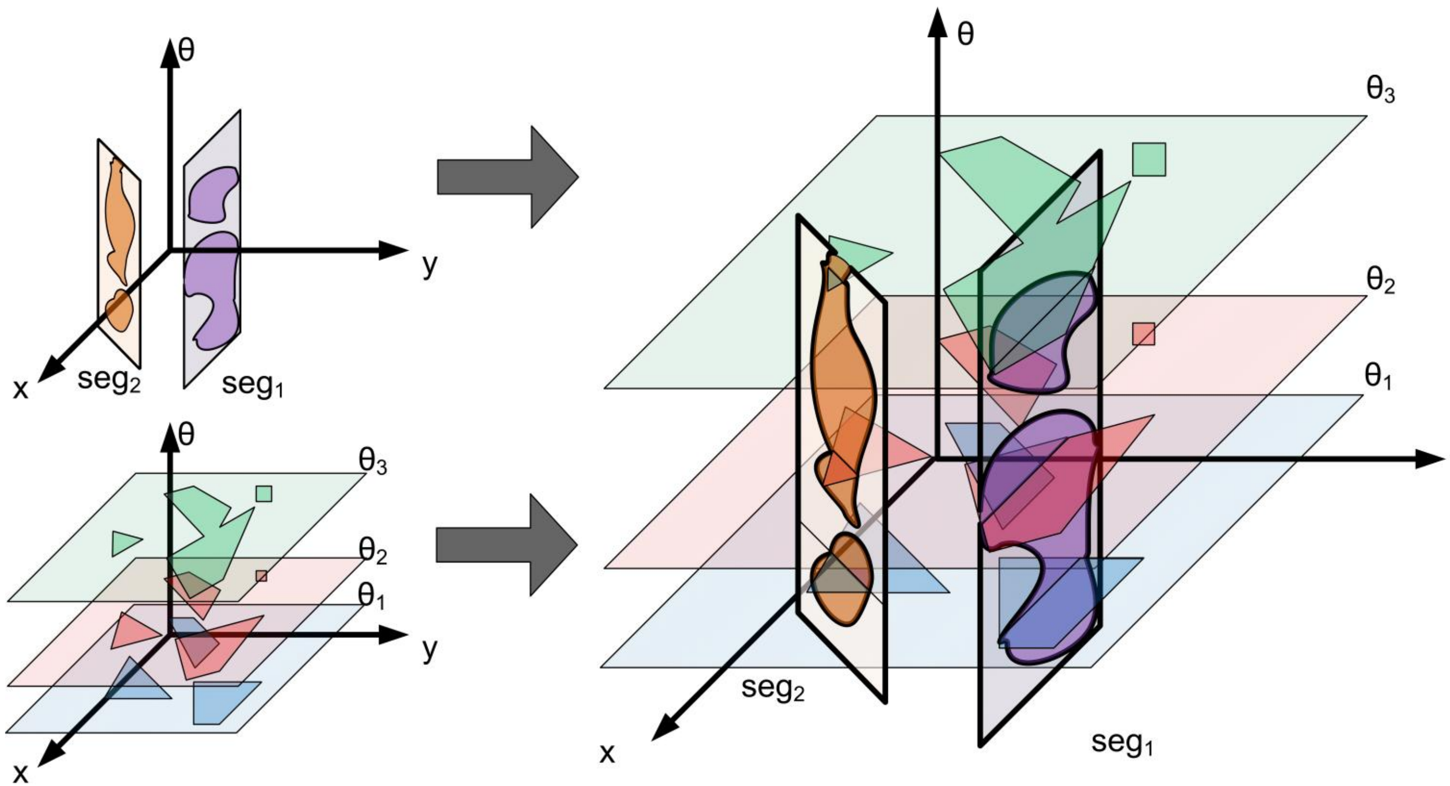
- Sampling-based multi-query planner
- Samples are entire manifolds of low dimensions
- Manifolds are **decomposed** analytically into cells
  - A cell in C_{free} is a **Free Space Cell** (FSC)





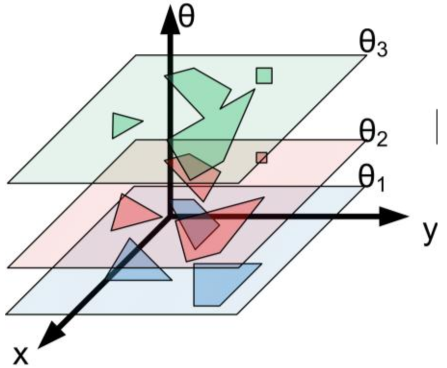
Overview

- **Preprocessing stage -** construct graph G = (V,E)
  - V – FSCs
  - E – Intersecting FSCs
- **Query stage**



Exploration Vs. Connection

- Manifold samples add
  - **vertices** (new connected components)



Exploration Vs. Connection

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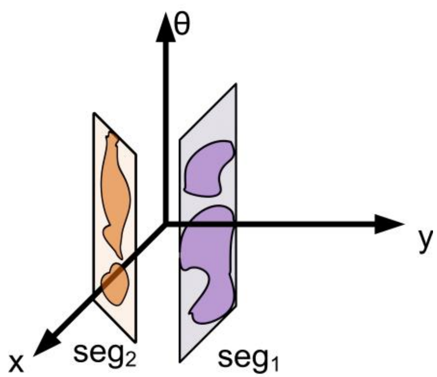

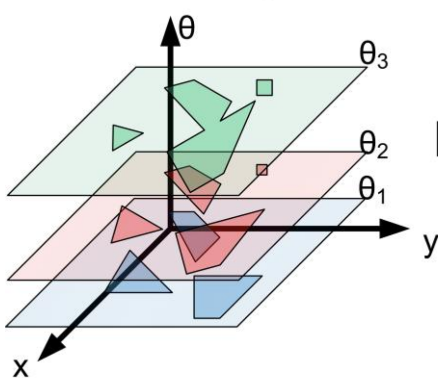

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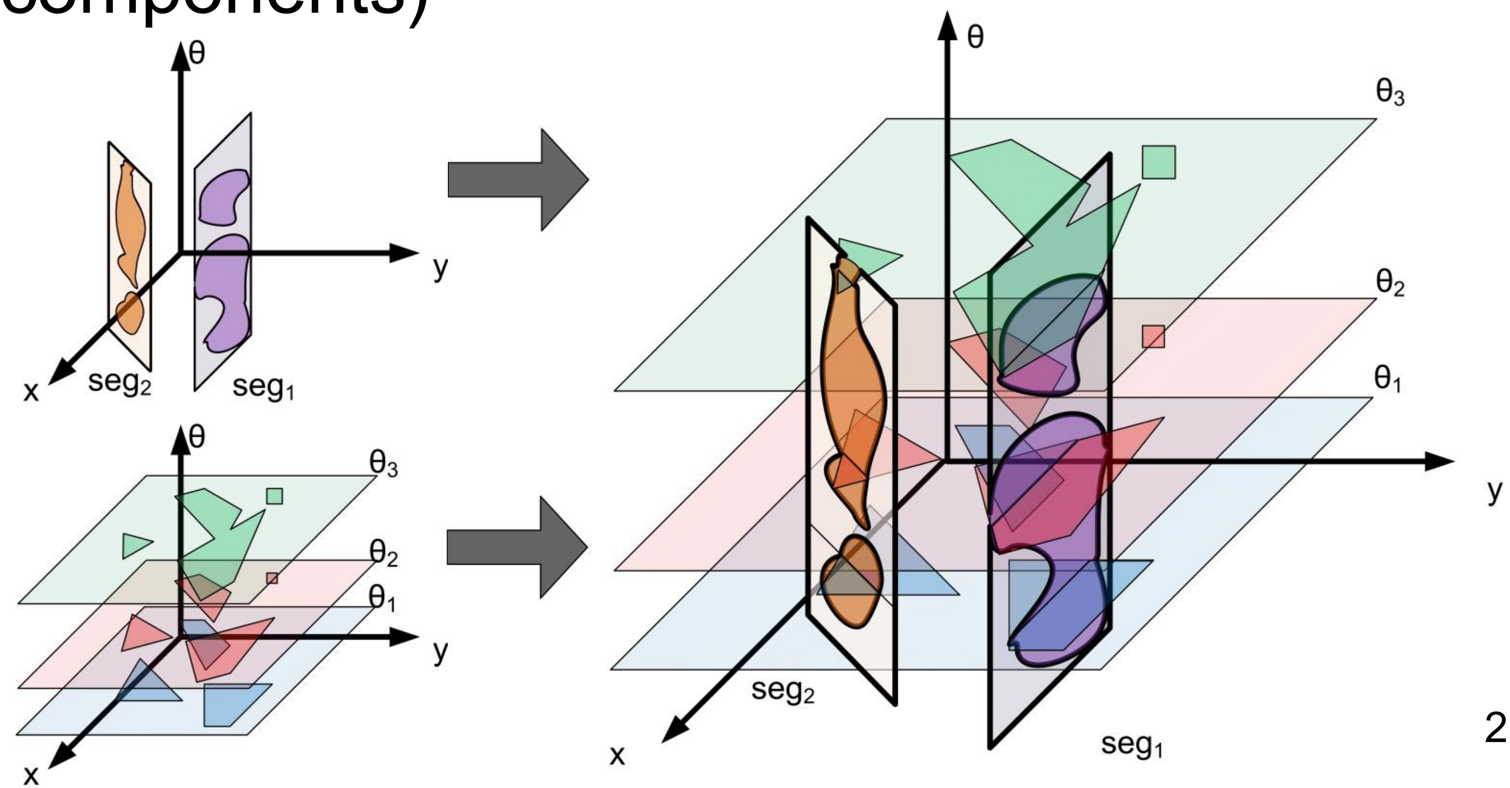
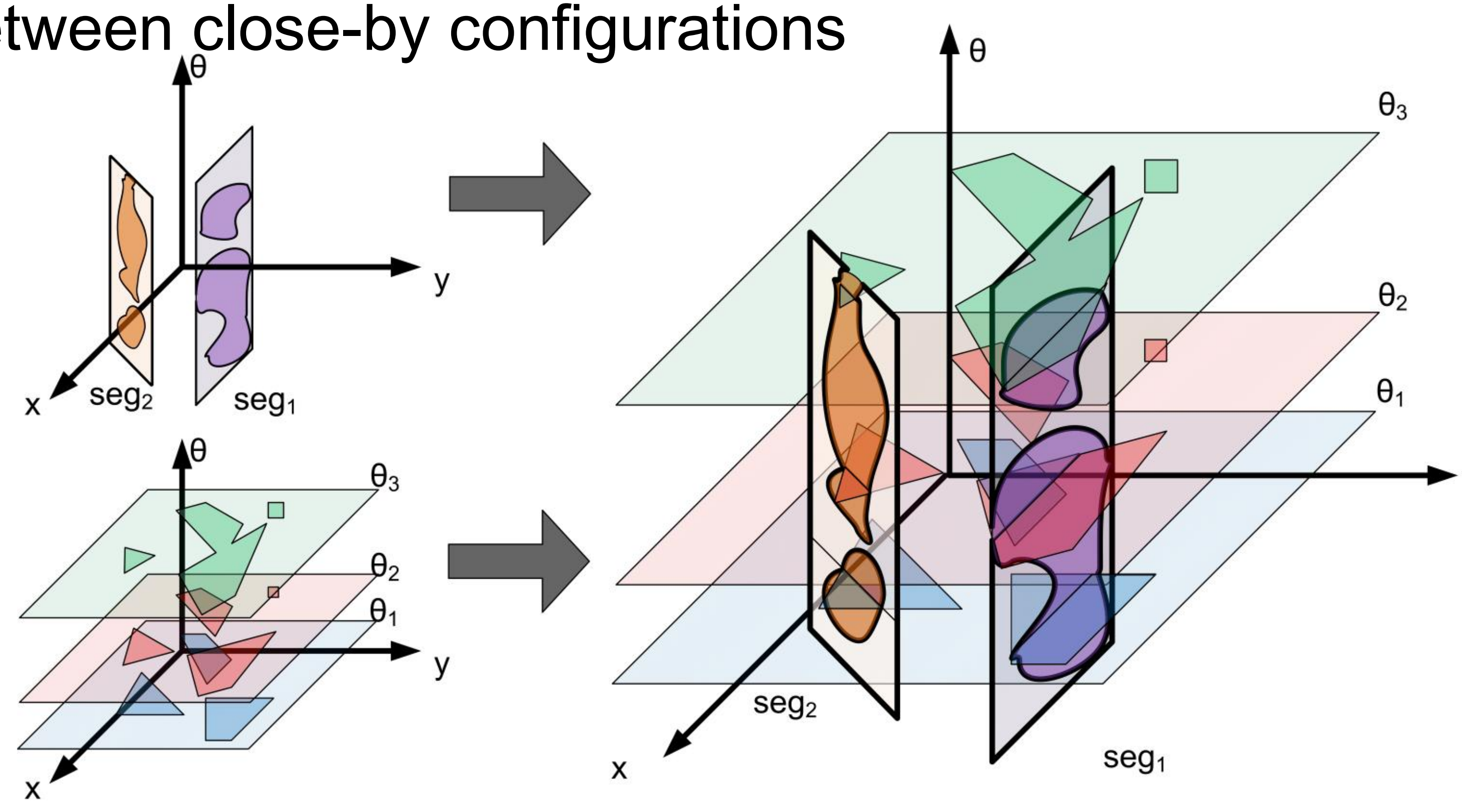


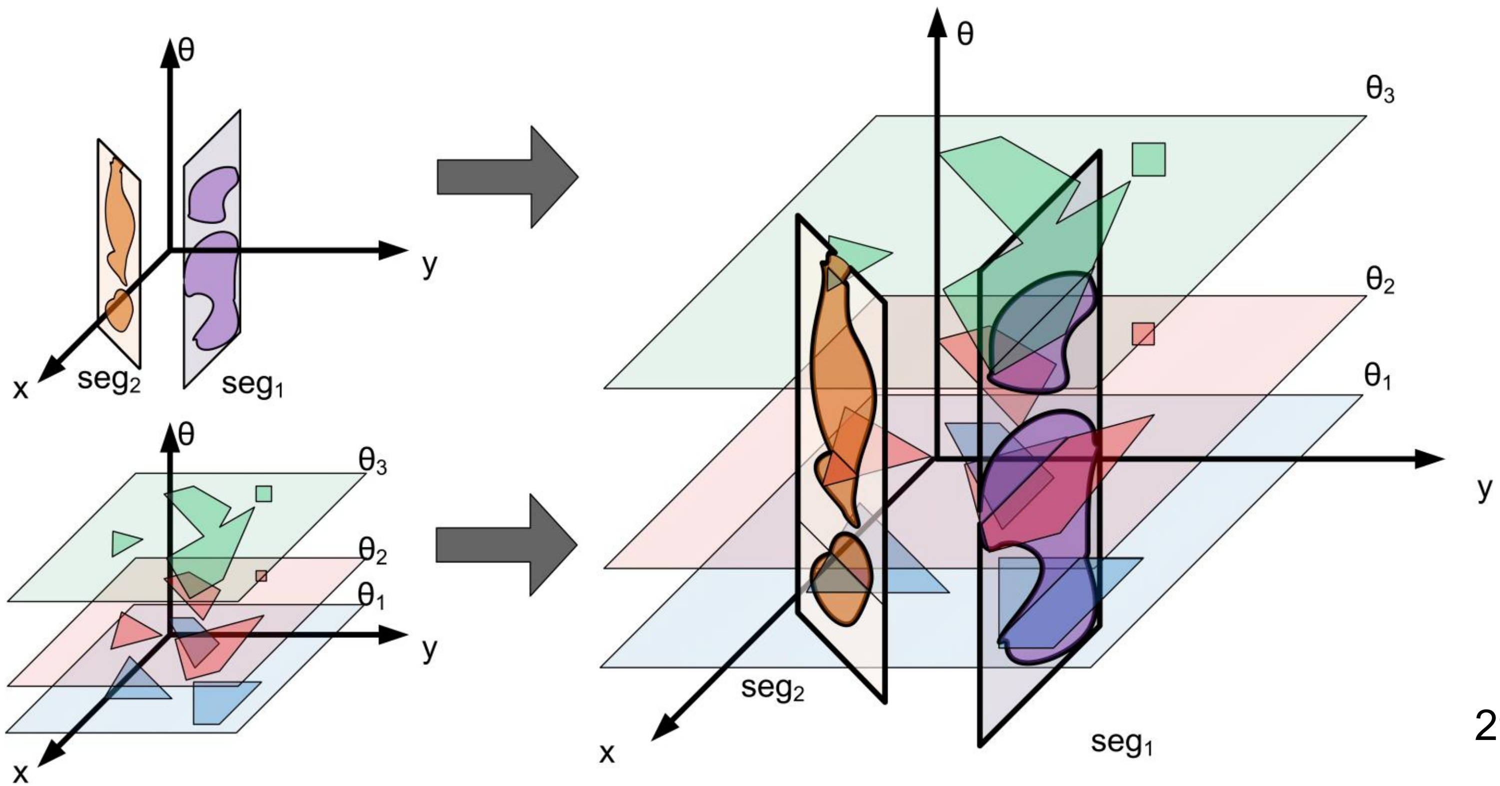
Desired Properties of Manifolds

- **Simplicity**: Easy representation, construction and decomposition
- **Covering**: Manifolds should be dense
- **Spanning**: Manifolds should allow local connections between close-by configurations



Comparison With PRM

	PRM	MMS
Sample type	Point	Manifold
Decomposition	Collision detector	Analytic primitive
Node type	Point	FSC
Node connection	Interpolation	Analytic (intersection)
Data structure	Roadmap graph	Geometric intersection graph of FSCs



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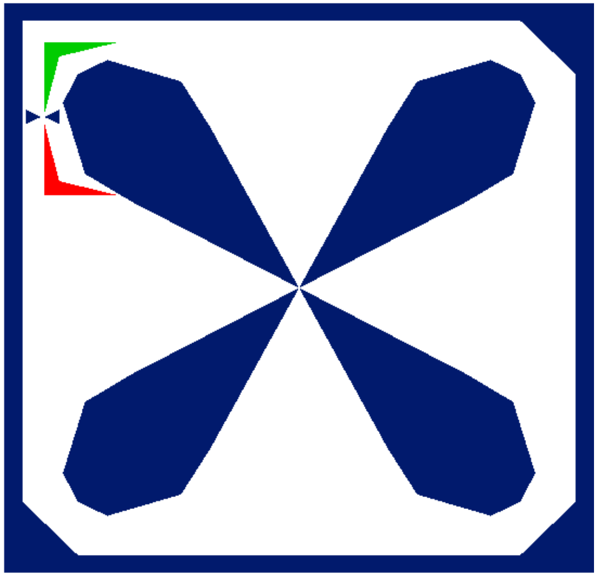
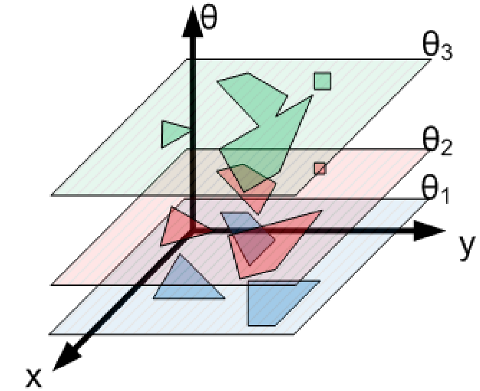
The Setting

- Two-dimensional polygonal robot R
- Three-dimensional configuration space
  - translation and rotation



Families of Manifolds

- **Fixed rotation angle**
  - Horizontal planes
  - Computation via Minkowski sums,



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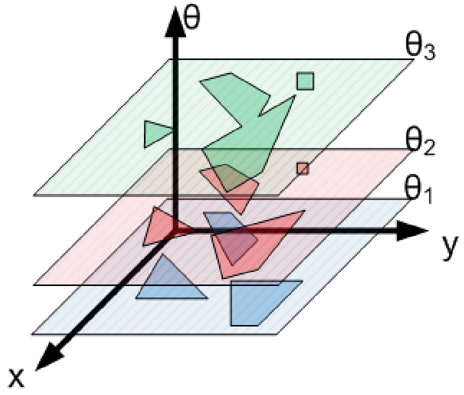
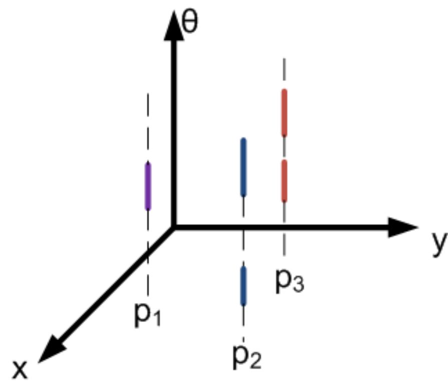


- **Fixed reference point**
  - Vertical lines
  - Computed analytically via critical angles



Fixed Rotation Angle (details)

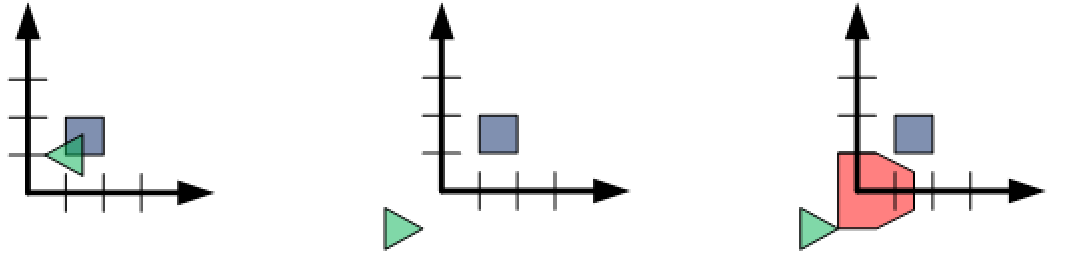
- (Thm) Let R(x,y) be a robot placed at x,y and P be an obstacle, then R intersects P iff

$$(x,y) \in P \oplus -R(0,0)$$

- If R(x,y) intersects P
  - Let q be the intersection point
  - $q \in R(x,y) \Rightarrow q - (x,y) \in R(0,0) \Rightarrow -q+(x,y) \in -R(0,0)$
  - $q \in P$
  - Thus, $(x,y) \in P \oplus -R(0,0)$
- If $(x,y) \in P \oplus R(0,0)$
  - There are points $r \in R(0,0)$, $p \in P$ s.t.
  - $(x,y) = p-r \Rightarrow p = (x,y) + r$
  - =>The robot placed at (x,y) intersects P

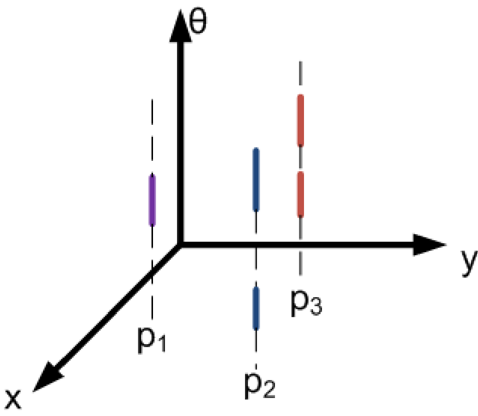
Fixed Rotation Angle (details)

- For a robot R with its reference point at the origin and an obstacle O, the forbidden space is represented by $-R \oplus O$



Fixed Reference Point (details)

- Parameterization:
  - $\alpha \in [0, 1]$ reference point on segment



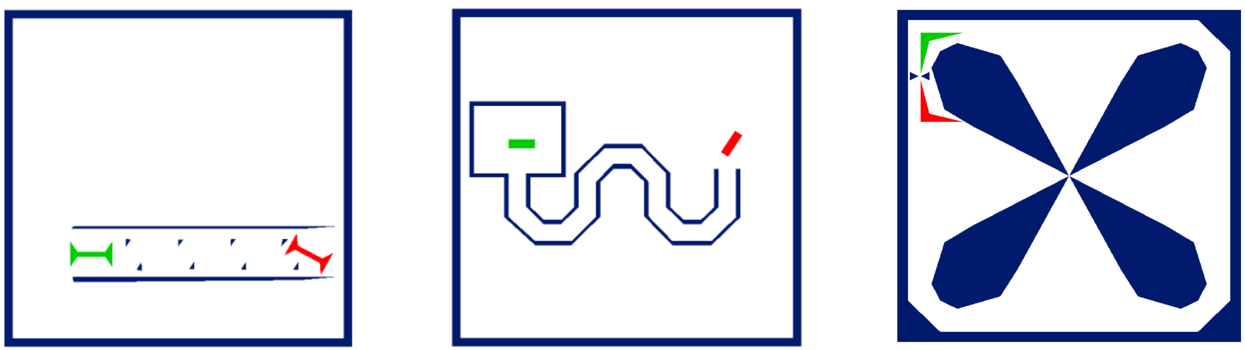
- Parameterized **critical angles** are in the form of algebraic numbers*

*Algebraic number - a number that is a root of a non-zero polynomial in one variable with rational coefficients

Experimental Results

- Scenarios

Tunnel Snake Flower

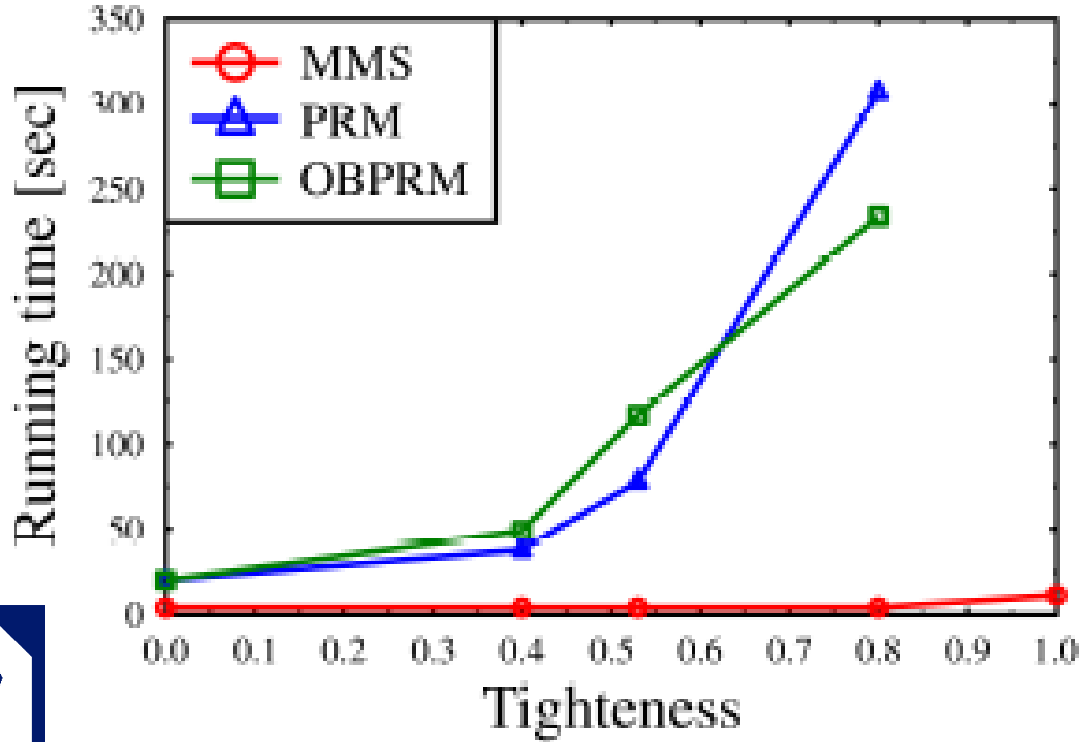


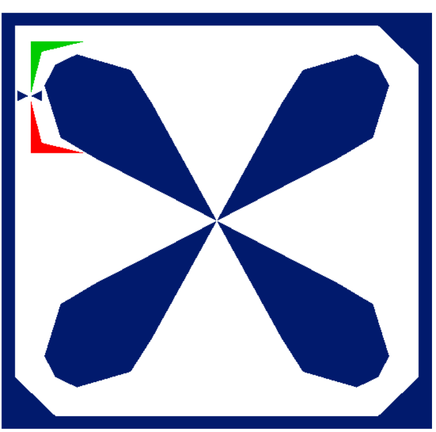
- OOPSMP PRM Implementation

Scenario	MMS			PRM			OBPRM		
	n_θ	n_x	t[sec]	k	res	t[sec]	k	res	t[sec]
Tunnel	20	128	**21**	10	0.005	**114**	10	0.01	**134**
Snake	40	256	**35**	10	0.02	**264**	10	0.01	**247**
Flower	40	256	**4**	20	0.02	**20**	14	0.01	**20**

Experimental Results

- Tightening the configuration space





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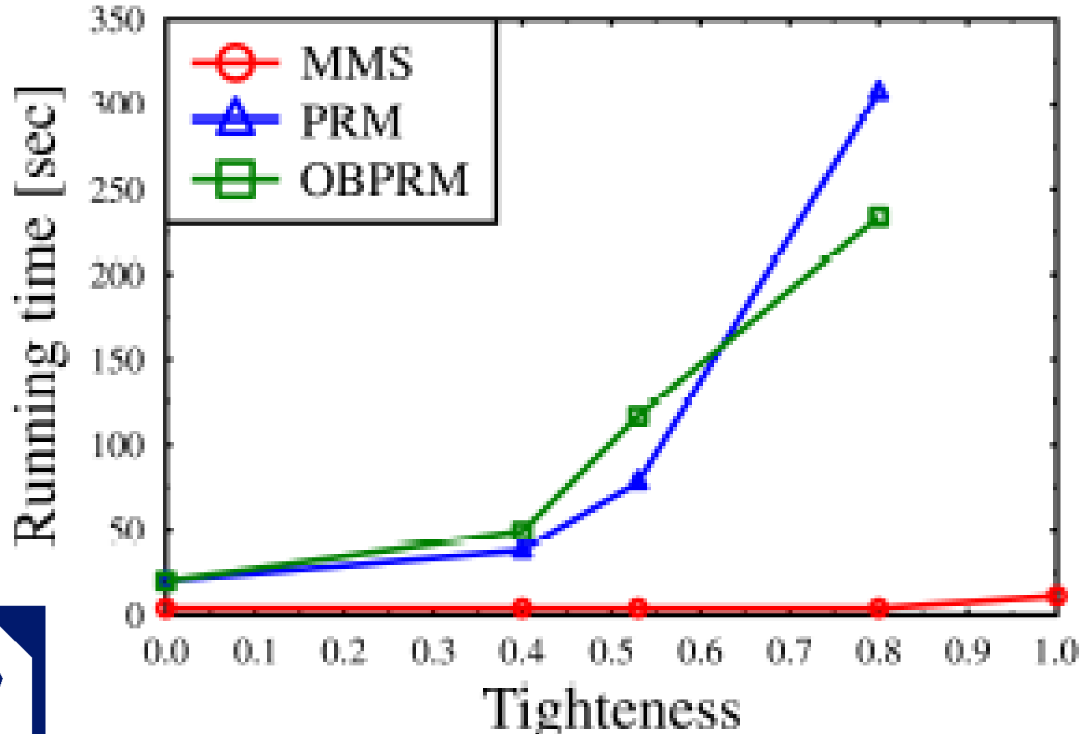

20-fold speedup