Applied Art Gallery Problems: CGAL meets CPLEX

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joint work with Tobias Baumgartner, Sándor P. Fekete, Mahdi Moeini, and Christiane Schmidt
Problems & Applications – Procedure – Variants – Numbers – Improvements
**Thm** [Chvátal ’75] (simple polygons): $n$ vertices. $\lfloor n/3 \rfloor$ guards are sometimes necessary and always sufficient.

**Thm** [Bjorling-Sachs, Souvaine ’95] ($h$ holes): same with $\lfloor (n + h)/3 \rfloor$ guards.
Hardness

**NP-hard**
- point guards with holes [O’Rourke & Supowit 1983]
- vertex guards w/o holes [Lee & Lin 1986]
- point guards w/o holes [Aggarwal 1986]

**APX-hard** [Eidenbenz, Stamm & Widmayer 2001]
Open Problem [O’Rourke ’05]: „Realistic light“ with fading intensity.
Art Gallery #3

inmetris3D GmbH
Braunschweig

Z
Fading (lin.)
Range
Workspace
Incid. Angle
Var. Density
Art Gallery #4

Fading (lin.)
Range
Workspace
Angle Guards

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Four Variants

- B
  - Fading (quad.)

- B/R
  - Fading (lin.)
  - Range
  - Workspace
  - Incid. Angle
  - Var. Density

- Z
  - Fading (lin.)
  - Range
  - Workspace
  - Angle Guards

- B
  - Fading (lin.)
  - Range
  - Workspace
Problems & Applications – **Procedure** – Variants – Numbers – Improvements
Ingredients of the Algorithm

\[
\begin{align*}
\min & \sum_{g \in P} x_g \\
\text{s.t.} & \sum_{g \in \mathcal{N}(w)} x_g \geq 1 \quad \forall w \in \mathcal{W} \\
& \mathbf{0} \leq x \leq 1 \\
& \forall g \in \mathcal{G}
\end{align*}
\]

Issues:

1. Cannot solve infinite IPs
   Solution: Solve LP relaxation (lower bound)

2. Cannot solve infinite LPs
   Solution: Restrict to finite sets (separation / column generation)
   \( \mathcal{G} \) is set of guard candidates, \( \mathcal{W} \) is set of witness points.
\[ \text{min} \sum_{g \in G} x_g \]
\[ \text{s.t.} \sum_{g \in G \cap \mathcal{V}(w)} x_g \geq 1 \quad \forall w \in W \]

Primal separation problem:
\[ \exists w \in P \setminus W : \sum_{g \in G \cap \mathcal{V}(w)} x_g < 1 ? \]
Dual separation problem:
\[ \exists g \in P \setminus G : \sum_{w \in W \cap V(g)} y_w > 1? \]
Algorithm

1. Pick initial $G$ and $W$
2. REPEAT
   i) Solve LP relaxation
   ii) Solve primal and/or dual separation
   iii) Update lower bound / upper bound / integer solution
   iv) Update $G$ and $W$
UNTIL gap is 0
3. Output solution

**Thm** [K., Baumgartner, Fekete, Schmidt JEA’12, ALENEX’10]
If terminates, then with optimal result.
Primal separation: Minimum coverage always attained in a face

But: Leads to shadow creep
Problems & Applications – Procedure – **Variants** – Numbers – Improvements
(Semi-) Easy Variants

**Var. Density**

RHS in LP formulation

\[ \min \sum_{g \in P} x_g \]

subject to

\[ \sum_{g \in V(w)} x_g \geq C_w \quad \forall w \in W \]

\[ 0 \leq x_g \leq 1 \quad \forall g \in G \]

**Workspace**

Intersect with workspace in dual separation

**Incid. Angle**

Intersect with visibility with double cone in dual separation

**B**

Solve LP as
Fading Model

\[ d = \|g - w\|_2 \]

\[ w \in \mathcal{V}(g) \]

sends with energy \( x \)

\[ \varrho(g, w) := \min\{d^{-\alpha}, 1\} \]

receives \( x \varrho(g, w) \)

(additive!)

If no capping at 1: no solution can exist!
Approximate $\varrho$

Given $\varepsilon$, approximate $\varrho$ with $\tau(g, w) := \|\varrho(g, w)\|_{1+\varepsilon}$

Properties: 1. $\frac{1}{1 + \varepsilon} \varrho < \tau \leq \varrho$

2. is a step function with only $O(\log(\text{diam}P))$ steps needed
Approximation der Separation

\[ \tau(g, w) := \| \rho(g, w) \|_{1+\varepsilon} \]
Approximation

**Thm** [K., Schmidt EuroCG’12]: $(1 + \varepsilon)$-Apx.
for primal and dual separation,
in time $O(|G|^2 (n + \log(\text{diam}P))^2)$.

**Thm** [K., Schmidt EuroCG’12]:
With fading & range: If terminates, then
$(1 + \varepsilon)$-Approximation.
Problems & Applications – Procedure – Variants – **Numbers** – Improvements
Implementation

Working implementation, mostly for classic AGP

IBM ILOG CPLEX

• CPLEX Callable Library
• LP- and IP-Solver

CGAL
Computational Geometry Algorithms Library

• Geometry parts
• Arrangement_2 package
• libGMP exact arithmetic
### Optimality Rates

<table>
<thead>
<tr>
<th>$n$</th>
<th>60</th>
<th>100</th>
<th>200</th>
<th>500</th>
</tr>
</thead>
<tbody>
<tr>
<td>Orth.</td>
<td>90 %</td>
<td>71 %</td>
<td>44 %</td>
<td>15 %</td>
</tr>
<tr>
<td>Simple</td>
<td>83 %</td>
<td>82 %</td>
<td>74 %</td>
<td>48 %</td>
</tr>
<tr>
<td>Koch</td>
<td>87 %</td>
<td>89 %</td>
<td>93 %</td>
<td>81 %</td>
</tr>
<tr>
<td>Spikes</td>
<td>100 %</td>
<td>100 %</td>
<td>100 %</td>
<td>100 %</td>
</tr>
<tr>
<td>Total</td>
<td>90 %</td>
<td>86 %</td>
<td>78 %</td>
<td>61 %</td>
</tr>
</tbody>
</table>

[K., Baumgartner, Fekete, Schmidt JEA’12]

But: always finds bounds

CGAL 3.4, CPLEX 12.1, Linux, Core2Duo 3.0 GHz, x86-64, 2 GB RAM
Not Closing the Gap

500 vertices
Typical Run

500 vertices

Lower
LP/IP
Upper

23s
220s
Gap (200 Vertices)

CGAL 4.0, CPLEX 12.1, MacOS 10.7, Intel i7 3.6 GHz, 16 GB RAM
Gap (500 Vertices)

CGAL 4.0, CPLEX 12.1, MacOS 10.7, Intel i7 3.6 GHz, 16 GB RAM
A Bad Example
A Bad Example
Problems & Applications – Procedure – Variants – Numbers – **Improvements**
Improvement Idea: Speed it up!

1. Compute visibility polygons
2. Guard/witness membership

Get rid of exact arithmetic! (but how?)
Better Visibility Algorithm (contribution Winnie Hellmann)

Speed up visibility polygons:
Algorithm with $O(n^2)$ preprocessing, $O(n)$ query [Asano, Asano, Guibas, Hershberger & Imai 1986].

Total runtime goes up (a bit)! (constants matter)
All nontrivial facets of $\text{conv}(\text{AGP}(G, W))$
with coefficients in $\{0, 1, 2\}$:

$$\sum_{g \in J_2} 2x_g + \sum_{g \in J_1} x_g \geq 2$$

where

$G' := \{g_1, \ldots, g_k\} \subseteq G$

$W' := \{w_1, \ldots, w_k\} \subseteq W$

$w_i \in \mathcal{V}(g_j) \iff i \neq j$

$J_1 := \{g \in G : 1 \leq |W' \cap \mathcal{V}(g)| \leq k - 1\}$

$J_2 := \{g \in G : |W' \cap \mathcal{V}(g)| = k\}$

+ other cutting planes...
Conclusions & Future Work

- Improve theory
- Improve implementation

Works!

You?
(We got funding!)

additional models
Thank You.